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A television series entitled "Interpreting Modern Mathematics" is sponsored by the Department of Education of San Diego County as an in-service educational service for teachers of grades kindergarten through six. The series is not intended to offer a comprehensive course in modern mathematics, but rather to deal with the content, materials, and methods of teaching the course. Programs consist of discussions and demonstration lessons followed by panel discussions of current issues. The content of the series is based upon the report of the California Advisory Committee on Mathematics which relates the essential strands of mathematics that should be incorporated in the elementary grades. The strands are: number systems and operations, geometry, measurement, applications of mathematics, functions and graphs, sets, the mathematical sentence, and logic.

A guide has been prepared to provide the teachers with resource materials and advance information about the programs in the television series. The first resource section contains background material and outlines of Programs 1-13. An abstract of some main features of the Advisory Committee's Report is also included. Other important sections of the guide present: an annotated bibliography of experimental programs and supplementary materials in modern mathematics; a list of recently published materials; selected references in modern mathematics; an annotated list of mathematics materials including selected films and filmstrips; and criteria for evaluating math textbooks.

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Television Mathematics Series

KFMB-TV, Channel 8, San Diego

INTERPRETING MODERN MATHEMATICS

Grades Kindergarten through Six

ED001384

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721 Capitol Avenue, Room 455
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INTERPRETING MODERN MATHEMATICS
Television Series

(For distribution to all teachers of grades kindergarten through six in the public schools of the San Diego area.)

Schedule of Televised Programs

To the Classroom Teacher

Section		Color of First Page of Section
1	Teacher's Guides for Programs 1 through 13	Salmon
2	Abstract of Some of the Main Features of the "Strands" Report of the Advisory Committee on Mathematics to the California State Curriculum Commission	Green
3	Experimental Programs and Supplementary Materials: An Annotated Bibliography	Buff
4	Recently Published Materials and Plans by Publishers for Publishing Materials Prior to 1965	Goldenrod
5	Annotated List of Mathematics Materials (Including Selected Lists of Films and Filmstrips)	Blue
6	Selected References in Modern Mathematics	Canary
7	Criteria for Evaluating Basic Textbooks in Mathematics for Grades One to Eight	Pink

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SCHEDULE OF TELEVISED PROGRAMS
Interpreting Modern Mathematics

Program	Date	Topic
1	October 6, 1962	Introduction to Sets Issue: What is Modern Mathematics?
2	October 13	More on Sets Issue: The Growth and Impact of Experimental Programs
3	October 20	Place and Base in Numeration Systems Issue: The Flexibility of Current Mathematics Materials
4	October 27	The Number System: The Number Line; Operations Issue: How Can Teachers Individualize Instruction in Mathematics?
5	November 3	Principles of Operations Issue: Is Modern Mathematics for Everyone?
6	November 10	Geometry Issue: How Do Teachers Get In-Service Help in Modern Mathematics?
7	November 17	Modern Mathematics in Local Classrooms Issue: How Can Teachers Relate Theory to Classroom Practice?
8	November 24	Rational Numbers Issue: What is the Role of the Department of Education, San Diego County, in Providing In-Service Help?
9	December 1	Mathematical Sentences Issue: How Can Teachers Evaluate Modern Mathematics Programs?
10	December 8	Applications of Mathematics (Problem Solving) Issue: What Has Been the Impact of the San Diego Four-Year NDEA Project in Elementary Mathematics?
11	December 15	Modern Mathematics in Local Classrooms Issue: How Do Teachers Obtain Mathematics Materials?
12	December 22	Modern Mathematics Materials Issue: What Criteria Are Used for Selecting Modern Mathematics Materials?
13	December 29	Textbook Selection and Future Plans

INTERPRETING MODERN MATHEMATICS

To the Classroom Teacher:

This guide provides you with resource materials and advance information about the programs in this series. The content of this series is based upon the "Strands" Report which the California Advisory Committee on Mathematics made to the California State Curriculum Commission in 1962. This report relates to the essential integrating strands of mathematics which the Advisory Committee recommends be incorporated in grades kindergarten through eight. The Committee recommends a mathematics program which focuses on basic concepts that unify mathematics. An abstract of this report is included for your information.

"Interpreting Modern Mathematics" is not intended to be a comprehensive course in modern mathematics. Rather it is designed to provide in-service education for teachers of grades kindergarten through six in that it deals with the content, materials, and methods of teaching modern mathematics.

Programs consist of discussions and demonstration lessons followed by panels dealing with current issues. The roles of the San Diego County Department of Education and other agencies which provide in-service help are stressed.

The Teacher's Guide which follows contains background information and outlines for Programs 1-13. Much of this background information has been taken from the "Strands" Report.

PROGRAM 1

INTRODUCTION TO SETS

Background Information

The concept of a set is fundamental for communicating ideas in mathematics, just as it is in our everyday language. We speak of herds, flocks, committees, armies, teams, groups of children, and so on. While it is conceivable that an adequate mathematical textbook could be written without using the word "set," it could not be written without using the concept.

We recommend that pupils become familiar with the simple concepts and language of sets. The approach should be from a linguistic point of view utilizing the role set terminology plays in accurate and unambiguous communication.

A word of caution: No mention of sets should be made unless an effective use is made of the terminology and concepts in the subsequent mathematical development. In particular, there is no place in the elementary school curriculum for the development of a "theory of sets" as an end in itself.

At the first level of introduction, which may be begun as early as kindergarten, the teacher uses the terms correctly and encourages their use by the children. There is no need to expect precise mastery by the child.

The teacher should talk of sets and their elements and use the convention of listing the elements of a set between braces, {...} . Arithmetic and geometry at all grade levels require extensive use of the basic concept of one-to-one correspondence, commonly called matching. This concept relies implicitly, if not explicitly, on a preconceived notion of set to describe what is being matched. The abstract nature of the number four can be learned by considering many different sets and by noting that the elements of some sets can be matched in a one-to-one correspondence with the elements of one set, {○, △, ▭, ★} , to which we assign the number four.

The early number program should provide many opportunities for children to differentiate relative sizes from among sets of two, three and four objects without resorting to counting. When four blocks are taken from a child's view and only three returned, he is likely to perceive that a smaller number have been returned, even though he is unable to count in terms of the number system. Through experiences of this kind he learns the language of comparison in number and measure--the same as, equals, as many as, more, many, fewer, bigger, longer, farther, thicker, fatter, et cetera. At this stage the concept of "more" has not yet been differentiated as between magnitude and multitude. Such experiences help form the needed background for emerging patterns of numeration and notation.

The child has had many preschool experiences with matching things one-to-one, two-to-two, and so on, and has been contrasting inequalities of sets of one, two, three and more. The next step in his learning is to associate the number names and symbols with sets of objects. The study of number relationships is promoted by arranging sets of objects (e.g., blocks) in sequential order, each set being one more than the preceding set. Experiences of this type help the child see the order behind our system of numeration and to associate the number words and symbols more easily with sets of objects. Because of the relative complexity of learning our sets of arbitrary counting numbers and numerals, a great deal of practice in a variety of situations is necessary if confusion resulting from half-learnings is to be avoided.

After counting with understanding has been achieved with both cardinal and ordinal numbers, children should learn how to write numerals. Writing beyond the numeral 9 should be accompanied with the concept of place value. As they count past nine they should either be taught or led to sense the principle of repetition in our system of numeration, even though they do not fully comprehend place value at the time. As they learn to write numbers beyond nine, the less refined notions of repetition must be clearly organized into concepts of place value.

Pupils should mature as rapidly as possible in conceiving of number as an abstraction and of a numeral as its symbol. A number is a mathematical concept; a numeral is a written symbol for a number and is the name of a number. The number 8 is the concept which the pupil associates with any set which has the property of containing eight members. Numerals are the first of the mathematical symbols which a pupil learns to use in the elementary school mathematics program.

In the final developmental stage of the language of sets in grades kindergarten through eight, we expect the pupil to use these set concepts and terms correctly and with reasonable precision. They will play an important role as he studies inequalities, solution sets, informal geometry and the application of mathematics to the social world, the technological world, and to mathematics itself.

Program Outline

I. Introduction to Series

II. Discussion of Modern Mathematics; Sets

III. Demonstration Lesson 1 - Sets (Pre-Number Activities)

Materials Used: Sets (for children) candies (9)
pennies (9)
blocks (8)
banana, doll, vase (3)
Flannel board
4 paper ducks and 2 rabbits for flannel board
4 pieces of paper (with names)
4 crayons

I. A. Define Terminology.

1. Set: group, collection, et cetera of objects
    a. Use in context -- set of blocks, dishes, boys, TV set, etc.
    b. Specify by describing.
    c. Call set by many names.

2. Members: objects belonging to a set
    a. Use sets to show that members may be related or unrelated, that size of members may vary.
    b. Understand the empty set (the set with no members).
    c. Change spacing and order of set members to show that set does not change.

B. Pair Members of Sets (one-to-one correspondence).

1. Pair two sets that match ("as many as," "none left over").
2. Pair two sets ("more than").
3. Pair two sets ("not enough," "fewer than").

C. Order Sets.

1. Bring out idea of "more than" between two sets.
2. Arrange sets in order from smallest to largest.
3. Develop need for naming the ordered sets (number names).

IV. Panel Discussion of Demonstration Lesson

V. Discussion of Issue: What Is Modern Mathematics?

VI. Preview of Program 2: More on Sets

MORE ON SETS

Background Information

Infinite and Finite Sets. Sets that contain an unlimited number of members are called infinite sets. The set of natural numbers is an example because there is no limit as to how far one can count. There is always a number that is one more; there are infinitely many natural numbers. Sets that have a countable number of members are called finite sets. A set of digits, the symbols that we use in our decimal system of numeration, is a finite set because there are only ten digits {0, 1, 2, 3, 4, 5, 6, 7, 8, and 9} .

An infinite set cannot be listed in its entirety as a finite set can be listed, but the listing can be indicated {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...} . The use of the three dots (...) at the end of the listed numbers means and so on in the same manner. For example, a person reading the above list of natural numbers can continue with them as far as needed. When a finite set is listed the symbol of three dots is used in the following fashion: {a, b, c, d, ..., k} . Read, "The set of letters of the alphabet a through k."

Equal Sets. Two sets are called equal if each contains the same members as the other, that is, if the elements are identical. The set of even numbers {2, 4, 6, 8, ...} and the set of numbers that are divisible by two {2, 4, 6, 8, ...} are equal sets because the same members belong to both sets.

Equivalent Sets. Two sets are considered equivalent if they contain the same number of members. The set of even numbers and the set of odd numbers are equivalent sets because every even number can be matched in a one-to-one correspondence with every odd number. The set {a, b, c} and the set {d, e, f} are equivalent but not equal sets; the number of the first, however, is equal to the number of the second.

Membership in the Set. An object (physical object or abstract idea) is considered to be a member of a specific set if it belongs to that set. The symbol "3" is a member of the set of digits of the decimal place value system because it is one of the symbols that belongs to the set of symbols designated by the description "the set of digits that are used in writing the numerals of the decimal place value system." Members of a set are also called elements of the set.

Subsets. One set is called a subset of another set if all members of the first set are members of the second set. The set of odd numbers is a subset of the set of natural numbers because all of the odd numbers are also natural numbers. The empty set is considered a subset of every set. The empty set has one subset only, itself.

Universal Set. When operations are performed on sets, it is helpful to limit the possibilities for the selection of elements to be members of the set. The possibilities are limited by specifying as the universal set the set of all the elements that are to be considered.

Operations. There are a number of operations on sets. Three types of operations are described below.

Set Union. The union of two or more sets results in a new set. The union of two sets is obtained when the members of the two sets are combined to form a third set. If the set of even natural numbers {2, 4, 6, ...} is joined to the set of odd natural numbers {1, 3, 5, ...}, the result is the set of natural numbers {1, 2, 3, 4, 5, 6, ...} which contains all of the members of both of the original sets.

Complementation. Let's think of the set of pupils in a classroom and the subsets of boys and girls. If we remove the set of girls, we have left the set of boys. This set may be called the remainder set or the relative complement. The members of the set of boys are not members of the set of girls; the two sets are said to be disjoint, they have no members in common. Another example is the set of natural numbers. The subset of even natural numbers may be regarded as the complement of the subset of odd natural numbers. The odds are also the complement of the evens. If we remove either, the other is the remainder set. Of course, when we undo the removing operation by joining the two disjoint sets, we have the original set.

Intersection. The intersection of sets creates a new set. If two sets intersect, the members of the new set are the elements of the first two sets that are members of both sets. The intersection of the set of multiples of the number "3" {3, 6, 9, 12, 15, 18, 21, 24, ...} and the set of multiples of the number "4" {4, 8, 12, 16, 20, 24, ...} is the set {12, 24, 36, ...} --those numbers that are members of both the set of the multiples of 3 and the set of the multiples of 4.

The universal set for the selection of the intersection of the multiples of 3 with the set of multiples of 4 could be the set of natural numbers or the set of even numbers because all of the elements of the intersection are members of the set of natural numbers and also members of the set of even numbers.

The Empty Set. A set is called the empty set if no members belong to it. The set of natural numbers that are both even and odd is empty because there is no natural number that is both even and odd. A natural number must be either one or the other. Therefore, the intersection of the set of even numbers and the set of odd numbers is the empty set. Zero is the cardinal number of the empty set.

Man's Need for Zero. Amazing as it may seem, it took man a long time to arrive at the abstract idea of the number zero. Example: Count the windows in the room where you are sitting. With which number did you start your counting? Wasn't it the number "one," the first number of the natural or counting numbers? What did you have to assume before you could start counting with "one" and come out with the correct number of windows? Before you answer let me ask this question: How many race horses are there in your kitchen? Your answer was, no doubt, "none" or "zero." We must assume zero before we can start counting 1, 2, 3, 4, ... Man's need to describe none led him to the abstract idea of the number zero. The empty set is obviously less than sets of one or more elements. In order it comes before a set of one member. When we join zero to the set of natural numbers, we call this new set the whole numbers. The set of natural numbers $\{1, 2, 3, 4, \ldots\}$ is a subset of the set of whole numbers $\{0, 1, 2, 3, 4, \ldots\}$.

Note to Teachers: Check the handwriting chart in your classroom. Does zero appear in the correct order in relation to the other digits? Zero at the end of the sequence may lead to misconceptions on the part of the children. The order of the ten digits should be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Program Outline

I. Discussion of Sets (continued)

II. Demonstration Lesson 2 - More on Sets

Materials Used: Assorted small toys

A. Define Set Union and Addition.

1. Explain that number exists only as a concept, an abstraction; define with use of the one-to-one correspondence between equivalent sets.

2. Show addition by set union.

a. Use set of objects.
b. Use chart notation.

B. Illustrate Set Complementation, Subtraction and the Integers.

1. Define subtraction as the inverse of addition using set complementation.

a. Use set of objects.
b. Use chart notation.

2. Show subtracting a larger number from a smaller number.

a. Show $5 - 5 = 0$.

(1) With sets of objects
(2) By set notation on charts
(3) On "whole number" number line

b. Discuss problem $5 - 7 = ?$; show impossibility of solution with the set of whole numbers.

Program 2 cont'd.

C. Illustrate Intersection and Highest Common Factor.

  1. Reduce 15/21.

    a. Find sets of factors of 15 and 21.

    b. Find the intersection of two sets.

      (1) List factors by set notation.
      (2) Use Venn diagrams.

    c. Select largest member of intersection.

  2. Reduce 20/30 by same procedure.

III. Panel Discussion of Demonstration Lesson

IV. Discussion of Issue: The Growth and Impact of Experimental Program

V. Preview of Program 3: Place and Base in Numeration Systems

PLACE AND BASE IN NUMERATION SYSTEMS

Background Information

Place Value. Place value is the fundamental principle of organizing enumeration beyond nine. In the decimal system of enumeration the ten digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, can be placed in an infinite number of positions arranged according to ordered powers of base ten, such as hundredths, tenths, units, tens, hundreds, ..., to express any real number.

A child's concept of place value may have its beginning in preschool or kindergarten number experiences. The concept is refined and extended in the primary number program. From discovering that bundles of ten are counted in a similar fashion to single objects, he should quickly move to seeing one symbolic object in tens' place as being equivalent to ten of them in ones' place. He learns that the number 14 can be represented by 1 object in tens' place and 4 objects in ones' place, as on an abacus.

A succession of experiences to extend the concept follow rapidly as, for example, writing numerals for large numbers, carrying in addition and multiplication, and regrouping in subtraction. To aid in understanding the concept of place value, pupils should be taught to work with bases other than ten. Once generalized, the concept of place value is of invaluable service in such situations as computation with decimal fractions, per cents, rounding in approximate computation, exponents, logarithms, and scientific notation.

Systems of Numeration. Our system of numeration, the decimal system, is based on powers of ten. Thus 2783 is our shorthand notation for $2(10)^3 + 7(10)^2 + 8(10)^1 + 3(10)^0$. An understanding of the nature of this system, and of the computational algorithms which make use of the system, is probably best obtained by some computation in a different system of numeration. The numeration system with base five, for example, would involve only the digits, 0, 1, 2, 3, and 4. The number which in this system is denoted 231 would in expanded decimal notation be $2(5)^2 + 3(5)^1 + 1(5)^0 = 50 + 15 + 1 = 66$. We do not advocate any large amount of computation using different systems of numeration, nor do we insist that our students should, like certain computing machines, be expert in computation in the numeration system of base two. But some study using a base other than ten is recommended.

The system of Roman numerals can be used effectively to show its inadequacy as a system of numeration. Although the Romans computed with the abacus, their complicated system of numeration was a handicap.

Bases of five, two, eight and twelve can be taught with understanding in the elementary school program. The purpose is not primarily to teach pupils to compute in non-decimal bases, but rather to improve their understanding of the principles of base, place value, and operation in our decimal system.

Program Outline

I. Discussion of Place and Base in Numeration Systems

II. Demonstration Lesson 3 - Place and Base in Numeration Systems

- A. Place Value

  Materials Used: Counting frame
  Hand number line
  Abacus (open-end)
  Money (dimes)
  Monroe Educational Calculator

  1. Develop the Concept of Ten in Our Numeration System.
     - a. Group in sets of 10 and count the sets.
     - b. Relate sets of 10 to children's knowledge of dimes and cents.
     - c. Interpret meaning of the number names, 10, 20, 30,..., 100.
     - d. Use digits in two-place numerals.
     - e. Relate digits and numerals to the calculator and to the hand line.
     - f. Order the tens (pointing out similarity to order of the ones).
  2. Utilize an Abacus to Explain Place Value.
     - a. Pair objects with counters.
     - b. Let one counter stand for one set of 10.
     - c. Give abacus rods place value analogous to place value of numerals.

- B. Number Bases
  1. Introduce Another Number Base.
     - a. Review base ten.
       - (1) Illustrate place value.
       - (2) Explain the use of exponents.
     - b. Discuss base five.
       - (1) Illustrate place value.
       - (2) Use grouping with base five.
       - (3) Use number line in base five addition.

III. Panel Discussion of Demonstration Lesson

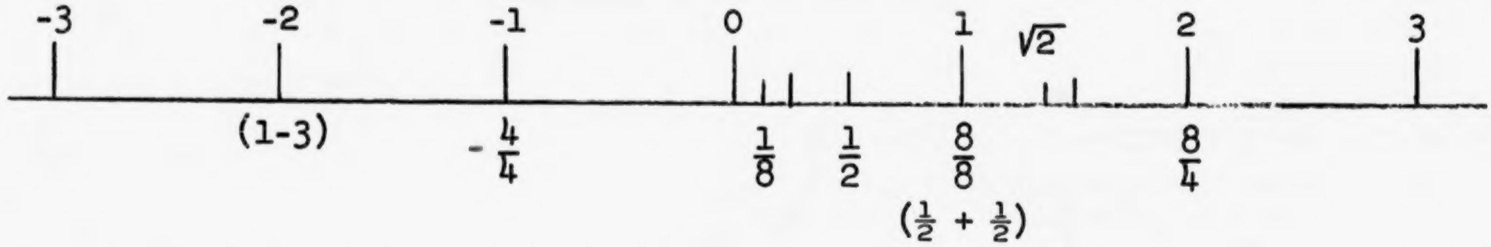
IV. Discussion of Issue: The Flexible Nature of Current Mathematics Materials

V. Preview of Program 4: The Number System

THE NUMBER SYSTEM

Background Information

Order: The Number Line. The ordering of the natural numbers should be emphasized from the beginning. The number line, or segments of the number line, should be introduced in preparation for later work. The use of the number line makes vivid many properties of numbers. A sketch, such as the one below, exhibits many important facts. By means of it,



one can visualize many names for the same number. There is no smallest fractional number, and there is no largest whole number. Numbers increase as we go to the right, and "less than" means simply "to the left of." Thus any negative number is to the left of 0, and is hence less than 0, and -4 is less than -1. And finally, this association of numbers with points on the number line serves as essential preparedness for the later study of the coordinate plane.

Principles of Operation. Much of arithmetic is concerned with the four fundamental operations. These are defined first for the whole numbers. Certain fundamental principles are emphasized which help to understand the extension of the operations to larger systems of numbers. Addition is initially defined in terms of counting (technically, in terms of counting the members of the union of two disjoint sets). Addition is an operation which assigns to a pair of numbers another number called the sum. Subtraction, on the other hand, may be defined in terms of addition; it is the operation for finding one addend if the sum and the other addend are known. The number 7-3 is that number which, when added to 3, yields 7. If this is clearly understood, many of the subtraction facts can be learned simultaneously with the addition facts. The "take away" interpretation of subtraction should not be used to an extent which obscures the fact that subtraction is the inverse operation of addition.

Division should be defined in terms of multiplication, just as subtraction is defined in terms of addition. The division facts can in large part be learned simultaneously with the corresponding multiplication facts. The intimate relationship between multiplication and division must be emphasized.

Closure. There are several principles which unify the development of the different number systems. Addition, subtraction, multiplication and division have in common the fact that they are operations, in the sense that operation with certain pairs of numbers results in assigning a number. This is an example of closure.

The sum and product of any two whole numbers $\{0, 1, 2, 3, \ldots\}$ are whole numbers. Hence the whole numbers are said to be "closed" under the operations of addition and multiplication. However, the whole numbers are not closed under the operations of subtraction and division; for example, the numbers resulting from $3 - 6$ and $2/5$ are not whole numbers. The rational numbers are closed under all four operations, with the single exception of division by zero.

Commutativity. Both addition and multiplication are commutative; that is, the order in which two whole numbers are added or multiplied does not affect the sum or the product. $2 + 3 = 3 + 2$. Commutativity should be emphasized, and the pupil should understand, by way of contrast, that subtraction and division are not commutative.

Associativity. Both addition and multiplication are associative; that is, the way in which the numbers are grouped for addition or for multiplication does not affect the sum or product. $(2 + 7) + 3 = 2 + (7 + 3) = 12$. By contrast, neither subtraction nor division is associative.

Distributivity. The distributivity of multiplication over addition is essential for arithmetic and fundamental to the ninth-grade teaching of factoring of polynomials. The principle of distributivity states that the product of a number and the sum of two numbers is the sum of the products of the first number and each of the addends. $2 \times (5 + 7) = (2 \times 5) + (2 \times 7) = 24$. Students should understand that all of the algorithms for multiplying whole numbers depend upon the distributive principle. The distributive principle also plays a fundamental role in computing the sum of two fractions having the same denominator.

Identity Elements -- Zero and One. The numbers 0 and 1 have special properties with respect to addition and multiplication: The addition of 0 to a number results in the same number, and so does multiplication of a number by 1. We say that 0 is the identity element with respect to addition and that 1 is the identity element with respect to multiplication. The major importance of the properties, and the reason they must be emphasized in the teaching of arithmetic, is their role in the more complicated study of rational numbers.

The foregoing principles upon which a number system must be built are essential for a sound mathematical foundation. They are as important to understanding mathematics as are the concepts of numeration, although the latter have been given much more attention in the elementary school curriculum. Although systems of numeration have been taught to pupils quite generally, there are additional experiences of learning about numeration that deepen pupils' knowledge and also serve to evaluate the quality of their generalizations.

The teacher should know and understand mathematical terms such as commutativity, associativity, function, and so on. However, such terminology should not be used prematurely with pupils. One would expect that by the end of the elementary mathematics program the pupil would be familiar with the standard mathematical terminology for simple arithmetic concepts.

Program Outlines

Program 4

I. Discussion of the Number System

II. Demonstration Lesson 4 - The Number Line; Operations

  A. The Number Line

    Materials Used: Number lines
    Slide from filmstrip, "The Number Line," Elementary Mathematics Series I, frame 8, Colonial Films

    1. Introduce number line as a geometric approach to development of number concept.

      a. Match whole numbers with equidistant points along a line.

      b. Start at zero, continue infinitely; use arrow or ... to indicate "and so on in the same manner."

      c. Arrange numbers in counting order.

        (1) Increase in value to the right.
        (2) Decrease in value to the left.

      d. Relate sets, hand line, and numerals to the number line.

      e. Relate numbers to the tens (8 is 2 less than 10).

      f. Compare values on the line ("greater than," "less than").

      g. Locate numbers on the line ("to the right of," "to the left of," "in between," "before," "after").

      h. Introduce sign for more than >.

    2. Discuss kinds of number lines; show how direction may vary.

  B. Operations

    Materials Used: Magnetic board with counters
    Stern blocks and number track

    1. Demonstrate commutative principle.

      a. Change the order of joining a pair of sets to show that interchanging the sets does not change the result.

        (1) Interchange pair of colored Stern blocks.

(2) Illustrate this interchange on a number line.

$$3 + 5 = 8$$
$$5 + 3 = 8$$
$$3 + 5 = 5 + 3$$

2. Demonstrate associative principle.

   a. Show how several sets may be joined to form a new set and how joining may be done in different ways and still give the same result.

   b. Interchange three sets of Stern blocks.

III. Panel Discussion of Demonstration Lesson

IV. Discussion of Issue: How Can Teachers Individualize the Instructional Program in Mathematics?

V. Preview of Program 5: Principles of Operation

Program 5

I. Discussion of the Number System (continued)

II. Demonstration Lesson 5 - Principles of Operation

A. Illustrate the problem 43 x 7.

1. Define multiplication as repeated addition or a "short-cut" addition.

2. Illustrate principles underlying solution.

   a. Closure principle of addition
   b. Distributive principle
   c. Closure principle of multiplication

B. Solve problem 3/4 x 5/7.

1. Define rational numbers.

2. Explain principles and definitions underlying solutions.

   a. Division as the inverse of multiplication
   b. Associative principle of multiplication
   c. Commutative principle of multiplication
   d. Closure principle of multiplication

III. Panel Discussion of Demonstration Lesson

IV. Discussion of Issue: Is Modern Mathematics for Everyone?

V. Preview of Program 6: Geometry

PROGRAM 6

GEOMETRY

Background Information

It has been recommended that more attention be given to early and progressive development of geometrical concepts in the elementary grades.

The introductory stage consists of becoming acquainted informally with some standard geometric shapes and forms. These would include such plane shapes as triangles, rectangles, squares, circles and ellipses; and such solid forms as spheres, cubes, rectangular prisms (boxes), cones and cylinders. Familiarity with these shapes and forms is gained through pictures, models and examples from everyday life. Some properties of these geometric figures may be informally developed so that the figures can be identified.

Following this recognition and identification level, the analytical stage begins. Some basic non-metric concepts concerning points, lines and planes are necessary for analyzing properties of geometric figures. A few of these concepts are presented in their intuitive setting. A point is conceived as a fixed location or position, having no size and represented by a small dot. A line (straight line) may be represented by a tightly stretched string or a tracing of a pencil along a straight-edge. A line extends indefinitely in both directions. A line is a set of points and is uniquely identified by any two of its points. A line segment is a subset of a line and is formed by two endpoints and all points in between. A ray is a subset of a line having a starting point and all points on one side of the starting point.

A plane is conceived as a flat surface that extends outward indefinitely, and which contains many figures (sets of points). A simple closed figure can be traced by starting at a point and coming back to it without crossing the path. Polygons are simple closed figures that consist of line segments. An angle consists of two rays which have a common vertex. Finger tracing of models of various solids can be used to develop concepts of surface (flat and curved), face, edge and vertex. Faces intersect in edges and vertices are the intersection of edges.

A basic concept in geometry is that of congruence. Two geometrical figures are said to be congruent if they have the same size and shape so that one of them can be superimposed exactly upon the other. Through exploratory activities pupils can be led to discover that line segments with equal measures are congruent and likewise angles with equal measures are congruent.

The basic geometric constructions using straight-edge, compass, protractor, and other instruments should be developed and applied to constructing some geometric figures with certain conditions stipulated. Some scale and perspective drawing using ruler and protractor should be introduced and the applications of geometry should be developed.

This is the basic content of informal geometry. It provides the necessary background for a more formal study of geometry later, or for the more specialized trade and vocational study which uses these geometric ideas.

One comment is necessary, however, concerning the sequential development of this strand. It is important that every opportunity be employed to connect geometry with arithmetic. The use of an array in discussing multiplication of numbers, for example, provides a geometric illustration of an arithmetic operation. More important, early and continued use of the number line in arithmetic provides the proper intuitive foundation for the eventual identification of the Euclidean line with the set of real numbers, the starting point of analytic geometry.

The pairing of geometric figures with numbers, which occurs in measurement and mensuration, involves the function concept. A line segment can be paired with a number which denotes its length. Linear measurement, then, is a process of determining what number is to be associated with a given line segment. Since many line segments have the same length, this is a many-to-one pairing. Geometric figures may be paired with numbers which may indicate their length, area, volume, perimeter, circumference, radius, or surface area as the case may be. All of these pairings are of the type that is basic to the idea of a function, and in particular to the idea of a set function since geometric figures are sets of points.

Program Outline

I. Discussion of Geometry

II. Demonstration Lesson 6 - Geometry

Materials Used: String

A. Introduction

B. Demonstration of an Intuitive Approach to Geometry Using String

1. Points

    a. Size
    b. Location of a point (A)
    c. Location of a second point (B)
    d. Relationship between points A and B

2. Lines

    a. The line through A and B
    b. Points on a line
    c. Length of a line
    d. Location of a third point (C)
        (1) On line AB
        (2) Off line AB

        e. Line through AC
            (1) When C is on AB
            (2) When C is not on AB
        f. Number of lines through a point

    3. Planes

        a. Figure determined by lines AB and AC
        b. Other lines on the plane
        c. Number of planes through a line
        d. Intersection of a line and a plane
        e. Intersection of planes

    4. Space

        a. Location of a point (D) off the plane
        b. Figure determined by points A, B, C, D
        c. Planes determined by points A, B, C, D
        d. Number of planes in space
        e. Skew lines
        f. Parallel lines
        g. Parallel planes

III. Panel Discussion of Demonstration Lesson

IV. Discussion of Issue: How Do Teachers Get In-Service Help in Modern Mathematics?

V. Preview of Program 7: Modern Mathematics in Local Classrooms

PROGRAM 7

MODERN MATHEMATICS IN LOCAL CLASSROOMS

Program Outline

I. Review of Content of Programs 1-6

   A. Sets

   B. The Number System: The Number Line; Operations

   C. Geometry

II. Presentation of Film Clips Showing Teachers and Pupils of the San Diego Area Applying Modern Mathematics

III. Discussion of Filmed Sequences

IV. Discussion of Issue: How Can Teachers Relate Theory to Classroom Practice?

V. Preview of Program 8: Rational Numbers

PROGRAM 8

RATIONAL NUMBERS

Background Information

The Rational Number System. In mathematics the expression "number system" has the technical meaning of a logical system of ideas about numbers consisting of principles (axioms), definitions and related concepts (theorems). The rational number system is studied in the elementary school program by means of certain sub-systems. First encountered is the system of natural (counting) numbers and zero {0, 1, 2, 3, ...} , sometimes referred to as whole numbers. These numbers are ordered, named in positional notation, and the basic operations and their properties are introduced. The system of fractions (positive rational numbers and zero), sometimes referred to as fractional numbers, is studied next. Such numbers as 0, 1/2, 2/3, 5/4, 1 3/8, 2 and all the numbers commonly referred to as common, proper and improper fractions; the natural numbers and zero; and mixed numbers are included in the system of rational numbers. The operations and principles developed for the natural numbers are extended to fractions. The negative rational numbers are introduced by first considering such numbers as -1, -2, -3, ..., and then later such numbers as -1/4, -7/8, -1 2/3, et cetera. The operations and principles are extended to include these numbers.

The rational numbers are closed under all four operations, with the single exception of division by zero. (3/0 does not exist, since there is no number which when multiplied by 0 produces 3.) The foregoing principles should be taught by the end of the K-8 mathematics curriculum.

Computation with fractions requires some knowledge of factors and multiples, and many opportunities for enrichment of the program can result from a study of some of the properties of these numbers. If one considers the set of natural numbers which are multiples of 4 and likewise the set of natural numbers which are multiples of 6, the sets are {4, 8, 12, 16, ...} and {6, 12, 18, ...} . The set of common multiples is {12, 24, ...} which is the intersection of the first two sets.

Certain key principles and concepts play an important part throughout the study of the rational number system, and in a very real sense, they unify the whole development. The arithmetic of the K-8 curriculum is a single mathematical structure and it must be taught in a sequential, developmental fashion. It must not appear to the pupil as a sequence of disconnected fragments or computational tricks.

Program Outline

I. Discussion of Rational Numbers

II. Demonstration Lesson 8 - Rational Numbers

  A. Explore the Nature of Mathematics.

    1. Need to solve physical problems

    2. Observation in the physical then translation to mathematics

  B. Explain the Set of Rational Numbers and Its Subsets.

    1. Natural numbers - the need to count

    2. Whole numbers

      a. The need for zero
      b. Addition and multiplication closed
      c. Subtraction and division not closed

    3. Integers

      a. The need for negative numbers
      b. Addition, subtraction, multiplication closed
      c. Division not closed

    4. Numbers and density

      a. Ordered relationship of integers
      b. Betweenness of numbers

    5. Rational numbers

      a. Points on the number line between the integers
      b. Division now closed
      c. Notation of rational numbers
        (1) An integer divided by an integer
        (2) Decimal notation extended

III. Panel Discussion of Demonstration Lesson

IV. Discussion of Issue: What Is the Role of the San Diego County Department of Education in Providing In-Service Help?

V. Preview of Program 9: Mathematical Sentences

MATHEMATICAL SENTENCES

Background Information

Since language is the primary means for the communication of ideas, a pupil should develop skills in using language to express ideas with clarity and precision. The language of mathematics is particularly suited to this purpose. The pupil's experiences with mathematical sentences should grow naturally out of his experiences with arithmetic. A beginning is made when a pupil recognizes and can write or say the numbers which tell how many objects there are in a set. The addition operation is conceived as the joining of two non-intersecting sets with a known number of objects in each and telling how many objects there are in the resulting set. If there are four members in one set and two members in a second non-intersecting set, the mathematical sentence is: four and two is equal to six. Many experiences with pictures and objects showing the joining of two sets will lead to the basic sentence form: ____ plus ____ equals ____, where pupils insert the numerals which tell the story. The symbols for plus, minus and equals are later used as replacements for words in this sentence form. Sentences involving all the operations are similarly developed through concrete or visualized experiences.

With this background a pupil learns that a mathematical sentence is simply a statement concerning numbers. A mathematical sentence may employ such nouns as number, set or point. Replacements for these nouns may be numerals or variables such as geometrically shaped frames or other symbols. The term "variable" need not be used with pupils in introductory stages and should not be used without adequate previous intuitive experience, although the teacher should have an understanding of the concept and use of a variable. A mathematical sentence also employs a verb form or its symbol such as is ($=$), is not ($\neq$), is less than ($<$) or is greater than ($>$). Conjunctions such as $+$, $-$, x, and $\div$ also occur and the noun-verb-noun sentence form is common.

The use of mathematical sentences in elementary instruction has, in the past, been handicapped by inadequate notation. If a sequence of various operations is involved, then symbols such as the various forms of parentheses, ( ), [ ] and { } are needed to specify the order of operations.

Thinking about a problem situation often results in the formulation of a mathematical sentence. When a pupil learns to describe a problem in his own words and then state the relationships by use of a mathematical model in sentence form, he has a sound approach to problem solving. The process of building a mathematical model should not be mechanical but should lead to a deep understanding of the mathematical process. Pupils who understand and use this approach to problem solving seldom ask, "Do I multiply or divide?" A problem situation may be described with a single statement or with several statements as a series of equations or inequalities.

Initially, mathematical sentence structure can be presented very simply. If the problem situation is one in which the pupil first adds and then subtracts, he can state the procedures in two separate sentences or combine them in a single sentence. Furthermore, a single mathematical sentence can be used to unify different problem situations. For example, the so-called three cases of per cent can be unified by the sentence $a \times 1/100 \times b = c$, where a represents the per cent, b the base, and c the percentage. We may solve for any unknown element in this sentence given the values of the other two elements.

Mathematical sentences are particularly useful in studying the properties of operations by providing a form for expressing such principles as commutativity, associativity, and distributivity. Sentences can also show the "doing" and "undoing" of inverse operations, as for example: $(8 + 3) - 3 = 8$ and $(24 \div 6) \times 6 = 24$. Sentences can provide a detailed analysis of certain operations involving fractions. For example: $5 + (2/3 + 1/2) = 5 + (4/6 + 3/6) = 5 + 7/6 = 5 + 1\ 1/6 = 6\ 1/6$.

Mathematical sentences may assist a pupil to make generalizations in the early stages of his mathematical education. If he observes that $0 + 1 = 1$; $0 + 2 = 2$; $0 + 3 = 3$; and so on, he should be able to generalize that for every whole number n, $0 + n = n$.

Much of a person's mathematical thinking involves equations and inequalities, so a clear understanding of the meaning of equality and inequality needs to be acquired. This and other questions of form of mathematical sentences are intimately related to logic.

It is very important that a textbook sequence provide a developmental arrangement of experiences explicitly designed to help the pupil acquire the ability to translate problem situations and facts into the language of mathematics and to acquire familiarity with mathematical sentences.

Program Outline

I. Discussion of Mathematical Sentences

II. Demonstration Lesson 9 - Mathematical Sentences

Materials Used: Stern blocks
Cuisenaire rods
Balance

A. Discover Mathematical Relationships with Stern Blocks, Cuisenaire Rods, and a Balance.

B. Translate Mathematical Relationships into Numerical Statements.

1. Show equalities.

2. Show inequalities.

    a. Demonstrate "not equal to."

    b. Demonstrate "greater than."

    c. Demonstrate "less than."

III. Panel Discussion of Demonstration Lesson

IV. Discussion of Issue: How Can Teachers Evaluate Modern Mathematics Programs?

V. Preview of Program 10: Applications of Mathematics (Problem Solving)

APPLICATIONS OF MATHEMATICS

Background Information

Applications are an essential part of the learning of mathematics in two important ways. First, it is through applications that the pupil is introduced to many mathematical concepts, and his understanding of these concepts is deepened as he applies them to a variety of situations. Second, interest and motivation increase as the pupil sees widespread significant uses of mathematics.

In the first growth stage of mathematical application, a mathematical concept is developed through varied experiences with specific objects and situations which are interesting to pupils. In the early grades, these will be found mostly in the immediate social environment of the pupils. Later the sciences, biological as well as physical, will provide experiences which lead to important mathematical concepts and will also provide new interpretations of previously developed concepts. Finally, it must not be overlooked that as pupils progress through the grades they acquire a fund of mathematical information which itself furnishes the raw material for still other important mathematical concepts.

As the pupil uses a concept in a variety of different concrete situations, its abstract nature gradually evolves. Growth in ability to make generalizations is very important and should not be neglected. Generalizations should not be formalized prematurely. The relationships between a concept and others previously developed and the relationships between concepts and computational algorithms should be observed.

A warning is needed here against confusing mathematical concepts with their interpretations. It is often said, for example, that there are two kinds of division: finding the size of a certain number of equal groups or finding the number of equal groups of a given size from a total number of objects. Actually there is but one division operation and these are but two of many interpretations which may be given to it. The fact that mathematical ideas are abstract and have no fixed interpretation is the reason for their utility in application.

In a later stage of growth in mathematical applications, the pupil is introduced to concrete problems whose solutions require interpretations of concepts. The pupil starts by observing and studying the problem situation carefully so that the key elements and relationships can be identified and translated into mathematical terms. For some situations this is an elementary task while for others more thought is required. In any case, the pupil should not be discouraged from rephrasing the problem situation in his own words.

Having translated the problem into mathematical terms, the pupil proceeds by using previously learned mathematical processes to find a numerical result or a desired formula, which should then be checked for reasonableness.

We recommend against a series of highly formalized problem-solving steps which the pupil can follow without reasoning or understanding. On the other hand, for most of the K-8 curriculum, problems should be sufficiently structured so that the interpretations are clear and some guidance is provided for the solution. This is the type of problem most commonly encountered in texts. A sufficient variety should be provided so that pupils can develop, informally, the beginnings of some of the more important problem-solving strategies.

Problems which arise in both childhood and adult experience are often unstructured. Here the pupil is called upon to do all the preliminary structuring; identifying the relevant aspects of the problem, making the necessary simplifying assumptions, gathering and organizing data and determining relationships using graphs or tables. Mathematical concepts are utilized through appropriate interpretations and are employed according to principles to find a solution. This is the most desirable form of mathematical application and should be encountered more often by pupils. We recommend that some problems of this type parallel the structured problems throughout the K-8 curriculum, with particular emphasis in grades 6-8.

The organizing of data involves some knowledge of statistical concepts and procedures. These processes and ideas are pointed toward organizing the data so that patterns and relationships can emerge. Therefore some study of statistical concepts should be included in the program.

In conclusion, a pitfall in using applications in a mathematics program needs to be mentioned briefly. This is the danger of becoming so involved in the details and the study of the application material that the development of mathematical understanding becomes subordinate. At the seventh and eighth grade levels, for example, there is the possibility of becoming so involved in the details of taxation or installment buying, which involve computations of a routine nature, that there is little advancement in mathematical understanding. Only a limited amount of time is available for the study of mathematics in school. A proper balance is needed between the part of this time devoted to applications and the part devoted to mathematics itself.

Program Outline

I. Discussion of Applications of Mathematics

II. Demonstration Lesson 10 - Problem Solving
    Select and solve problem showing that there are a variety of ways to solve it.

III. Development of Applications of Mathematics to Subject-Matter Areas

    A. Application of Mathematics to Social Studies

    B. Application of Mathematics to Science

    C. Application of Mathematics to Mathematics

IV. Panel Discussion of Sections II and III Above

V. Discussion of Issue: What Has Been the Impact of the San Diego Four-Year NDEA Project in Elementary Mathematics?

VI. Preview of Program 11: Modern Mathematics in Local Classrooms

PROGRAM 11

MODERN MATHEMATICS IN LOCAL CLASSROOMS

Program Outline

I. Review of Content of Lessons 8-10

   A. Rational Numbers

   B. Mathematical Sentences

   C. Applications of Mathematics (Problem Solving)

II. Presentation of Film Clips Showing Teachers and Pupils of the San Diego Area Applying Modern Mathematics

III. Discussion of Filmed Sequences

IV. Discussion of Issue: How Do Teachers Obtain Mathematics Materials?

V. Preview of Program 12: Modern Mathematics Materials

PROGRAM 12

MODERN MATHEMATICS MATERIALS

Program Outline

I. Introduction

II. Discussion and Demonstration of Materials to Improve Mathematics Learning

   A. Exploratory Materials

   B. Visual Materials

   C. Symbolic Materials

III. Summary

IV. Discussion of Issue: What Criteria Are Used for Selecting Modern Mathematics Materials?

V. Preview of Program 13: Textbook Selection and Future Plans

TEXTBOOK SELECTION AND FUTURE PLANS

Background Information

Schedule of Activities Involved in Adoption of Arithmetic Textbooks, Grades 1 to 8 (Present Adoption Period Ends June 30, 1965)

Statewide study of mathematics instruction	Began early in 1960, and is expected to continue through 1963. Summary of findings relating content and teaching methods for elementary school grades is expected to be available to State Curriculum Commission in early spring of 196
Formulation of criteria for evaluation of arithmetic textbooks (by subcommittee of State Curriculum Commission)	August, 1961 to May, 1962
Adoption of criteria by Curriculum Commission	May, 1962
Recommendation by Curriculum Commission for call for bids for arithmetic textbooks	May, 1962
Authorization for call for bids by State Board of Education	June or July, 1962
Issuance of call for bids	September, 1962
Submission of textbooks	February 1, 1963
Evaluation of books by Curriculum Commission	February to November, 1963
Recommendation by Curriculum Commission of books for adoption	About November 16, 1963
Adoption by State Board of Education	January, 1964
Printing of books or arranging for purchase	Following January, 1964
Beginning of adoption period	July 1, 1965

CALIFORNIA STATE DEPARTMENT OF EDUCATION
Bureau of Textbooks and Publications
February 28, 1961

Program Outline

I. Introduction to Textbook Selection and Roles of Educational Agencies

II. Discussion of Roles

   A. California State Curriculum Commission

      - Mrs. Jeannine Sevey, Member
        State Curriculum Commission
        Vice-Principal, South Oceanside School

   B. State Advisory Committee on Mathematics

      - Mrs. Leslie Beatty
        Director of Research and Curriculum Materials
        Chula Vista City School District

   C. Department of Education, San Diego County

      - Dr. R. Van Allen
        Director of Curriculum Coordination

   D. San Diego Unified School District

      - Dr. William H. Stegeman
        Assistant Superintendent
        Curricular Services Division

III. Panel Discussion of Educational Agency Roles with a Look to the Future

IV. Summary of Panel and Series

ABSTRACT OF SOME OF THE MAIN FEATURES OF THE "STRANDS" REPORT

OF THE

ADVISORY COMMITTEE ON MATHEMATICS TO THE CALIFORNIA STATE CURRICULUM COMM

April, 1962

ABSTRACT OF SOME OF THE MAIN FEATURES OF THE "STRANDS" REPORT OF THE ADVISORY COMMITTEE ON MATHEMATICS TO THE CALIFORNIA STATE CURRICULUM COMMISSION

April, 1962

I. Introduction

The basic topics in the K-8 curriculum are Number Systems, Geometry, Measurement, and Applications. But these alone are not enough. There have been remarkable advances in this century in the clarity and precision of mathematical discourse. We have come to see that many apparently diverse notions are special cases of a few underlying concepts. We believe it essential that mathematical instruction bring some of this economy of thought to the student. Consequently, we recommend a planned and conscious development of certain strands which contribute to this clarity, precision, and economy. These are the strands labeled Sets, Functions, Logic, and Mathematical Sentences.

A sound curriculum is not alone sufficient. Sound and meaningful concepts can be just as poorly taught as unsound concepts. A reorientation of the mathematical program which emphasizes structural aspects will be unsuccessful unless the pedagogy is successful.

II. Development of the Fundamental Concepts

A. Development of the number systems: The natural numbers, the whole numbers, the integers, the rational numbers, and an introduction to the real numbers

1. Matching or one-to-one correspondence as the basis for the child's instruction in learning to count

2. Definitions of the operations and their relationships

    a. The cartesian product concept of multiplication
    b. Subtraction as the inverse of addition
    c. Division as the inverse of multiplication

3. The properties of the operations such as: Commutativity, associativity, distributivity, closure, and identity in appropriate terminology for an elementary school child

4. The special properties of 0 and 1

5. Attention to the use of the terms "number" and "numeral"

6. The number line as a visualization for mathematical abstractions such as: The ordering and comparing of numbers, the association of numbers with points on a line, different numerals for the same number, no smallest or largest rational number, number facts, and the concept of the operations

7. Sufficient development of the multiplicative structure of the natural numbers, including the study of factoring, prime numbers, and tests of divisibility

8. Rate, ratio, and per cent presented as special interpretations of rational and real numbers

9. The rational numbers as periodic decimals and the introduction to the study of real numbers by showing that numbers like the $\sqrt{2}$ are not rational

B. Systems of Numeration and Notation

1. A thorough development of the place-value, decimal system of numeration and its use in computational algorithms

    a. A consideration of systems of numeration with different bases to provide a deeper understanding of the decimal systems

    b. A study of some historical non-positional numeration systems, for their vestiges of utility and for their value as contrasting with positional systems

2. A knowledge of integral powers of ten, the meaning of exponents, and scientific notation as a natural outgrowth of place value notation

C. Geometry

1. An introduction to the simpler plane configurations, considered as sets of points, including line, line segment, ray, angle, and closed figures

2. A discussion of the simpler relationships among geometric configurations, including intersection, parallelism, perpendicularity, congruence, and similarity

3. Simple geometric drawings using a variety of instruments

4. Recognition of common plane and solid figures and their distinguishing characteristics

5. Geometric relationships should be interpreted to yield arithmetic properties as in arrays, number line, graphs, and solutions of number sentences

D. Measurement

1. The concept of measurement as an association of a number with an object through comparison with an arbitrarily chosen unit

2. The common standard units, including those of the metric system

3. Conversion of measures to different units

4. Estimation

5. Computation with approximate data including the ideas of precision and accuracy

6. Drawing to scale

E. Functions and Graphs

1. In order to provide experiences which will contribute to the development of the abstract mathematical concept of function, the pairing of numbers with objects or other numbers should be emphasized in such topics as counting, lengths, areas, formulas, tabular and graphical data, and operations

F. The Mathematical Sentence (Used in statements of equality and inequality)

1. Mathematical sentences should be used in the presentation of number facts

2. Use of mathematical sentences in problem solving

3. Use of mathematical sentences as an aid in stating mathematical principles precisely

G. Logic

1. A correct use of equality; to be used as meaning logical identity

2. An informal treatment, at a level that has meaning for elementary school children, of such concepts of logical thinking as definition, implication, and quantification

H. Sets

1. The set concept, and related concepts, should be introduced informally and used effectively throughout to develop ideas of arithmetic and geometry

2. The terminology and notation of sets should be introduced and used only as appropriate

I. Applications and Problem Solving

1. A proper balance between mathematics, per se, and its applications

2. Significant applications in many subject matter areas, and the requisite statistical notions

3. The text should encourage students to translate problem situations into their own language and then into appropriate mathematical models and conversely

4. A variety of methods of solving problems should be encouraged

EXPERIMENTAL PROGRAMS AND SUPPLEMENTARY MATERIALS

An Annotated Bibliography

May 14, 1962

INTRODUCTION

The Advisory Committee on Mathematics to the California State Curriculum Commission has recommended a mathematics program which focuses on basic concepts that unify mathematics. The committee believes that insight into these structural concepts will facilitate understanding of mathematical reasoning, learning of mathematical skills, and the changes in skills which the future promises to require. A method of teaching which encourages conjecturing, hypothesizing and experimenting is considered important to effective instruction.

The program that is recommended for Kindergarten and Grades One through Eight includes the many familiar topics which have proved their enduring usefulness. These traditional topics are presented in relation to eight fundamental concepts, strands which run throughout the entire curriculum. The first four strands in the following list are regarded as the major strands for K-8. The others are included as they contribute to the clarity and precision of the pupil's thought.

Strands natural rational real integers

✓ 1. Number systems and operations, including numeration and computation Ass Com Dis. Numeration Writing numbers -

✓ 2. Geometry

✓ 3. Measurement

✓ 4. Applications of Mathematics

5. Functions and Graphs

SMSG * 6. Sets

SF * 7. The Mathematical Sentence *

8. Logic

A Subcommittee of the Advisory Committee was asked to study programs and materials in relation to the eight strands; one outcome of the study is this annotated bibliography. It is being distributed to school districts as a help to them in studying experimental programs and in finding supplementary materials for classroom use. These materials should assist teachers and pupils in making the transition from traditional arithmetic to the changes which will undoubtedly be found in new textbooks.

The publications which were reviewed fall into two categories, (1) materials from experimental programs and (2) materials specifically prepared for supplementary use by publishing companies. While some of the experimental programs are now nearing completion, the publications have been included in the experimental category.

EXPERIMENTAL PROGRAMS AND MATERIALS

The experimental programs have certain characteristics in common. Most of them accept such familiar goals of the traditional curriculum in arithmetic as understanding of the decimal place value system of notation, understanding of algorithms and development of skill in computation, and ability to apply mathematical knowledge in problem solving. To these goals are added understanding of the fundamental structure of mathematics and methods of mathematical reasoning. Certain concepts are introduced earlier than previously. In many of the programs there is emphasis on discovery and intuitive thinking in the teaching method. Often classroom teachers and mathematicians have cooperated in planning and preparing materials. In general, the materials have been introduced to teachers in carefully planned preparation programs. More in-service education is required for their use than is needed for use of materials prepared by publishers to supplement traditional programs.

1. Greater Cleveland Mathematics Program

The Educational Research Council of Greater Cleveland initiated GCMP. Teams of teachers, supervisors, administrators and consultants developed materials which were tried out in classrooms and then revised. An extensive in-service education program preceded general use in classrooms. The published materials provide a complete curriculum for Kindergarten through Grade Eight. Each of the strands receives thorough treatment; other material, not covered by the strands, is included. Teachers are encouraged to help children discover relationships.

Science Research Associates is publishing the tests, teacher guides, and some in-service education materials.

Publications available for purchase:	Cost:
Complete Review Set, Grades K, 1, 2, 3	$ 18.25
Administrator's Manual	1.75
Key Topics in Mathematics for Primary Teachers	1.50
Pads of worksheets (8" x 10") for K, 1, 2, 3 (20 or 30 sheets per pad)	Obtain from publisher

Source:

Science Research Associates, Inc.
259 East Erie Street
Chicago 11, Illinois

2. The Madison Project of Webster College and Syracuse University

This supplementary program which was begun in 1957 in the Madison Junior High School, Syracuse, New York, has been extended to elementary grades. Strands #1, Number Systems and Operations, and #7, The Mathematical Sentence are covered intensively; some aspects of Geometry, Measurement, Applications of Mathematics, and Functions and Graphs are included.

The approach taken by the author is summed up in the statement, "We explore the fascinating concepts of mathematics as long as they fascinate us; when they threaten to become routine, we turn to a new topic....After all, our purpose is to get students thinking, on their own, about the basic concepts of mathematics."

Publications available for purchase:	Cost:
Madison Project Workbooks with accompanying Teacher's Manual (mimeo)	$ 9.50 set of both
Tape recordings and 16 mm. sound films of project classes	Request information from Dr. Davis

Source:

Dr. Robert B. Davis
Mathematics Department
Syracuse University
Syracuse 10, New York

3. San Diego Metropolitan Education Study Council

A number of school districts in San Diego County are cooperating in a NDEA project to extend the arithmetic program for advanced pupils in Grades 3-6. As part of the project, six booklets, varying in length from 32 to 72 pages, were prepared by teams and published by the San Diego County Department of Education. These booklets are designed for use by advanced pupils with special ability in mathematics. The materials do not purport to develop complete understanding of an area but rather to provide an introduction or a different approach which will extend ideas and create new interests. The decimal place value system of numeration is given considerable attention. Measurement, Applications of Mathematics, and Graphs and Functions are included in one book or another. While the orientation is somewhat traditional, certain unifying ideas are emphasized. Understanding the Slide Rule does an excellent job of building the necessary rationale; however, it may be more suitable for use in Grades 7 and 8 than Grades 5 and 6 in many schools.

Direct inquiries concerning the project to:

Dr. Paul Zintgraff
San Diego County Department of Education
6401 Linda Vista Road
San Diego 11, California

Publications available for purchase:

Independent Learning Booklets--	Suggested Level	Cost:
We Travel with Money	Grades 3 and 4	$ 1.00
We Learn to Measure, We Measure to Learn	Grades 3 and 4	1.00
Mental Arithmetic in Subtraction and Division	Grades 5 and 6	1.00 (2)
Estimation, The Decimal Point in Division	Grades 5 and 6	
Probability and Statistics	Grades 5 and 6	1.00
Understanding the Slide Rule	Grades 5 and 6	1.00

A brief guide for teachers is in preparation.

Source:

Aztec Book Shop
San Diego State College
5402 College Avenue
San Diego 15, California

4. School Mathematics Study Group

The primary purpose of SMSG, which is supported financially by the National Science Foundation, is to foster research and development in the teaching of mathematics. An extensive experimental program was begun in 1959. The materials for classroom use were planned and written cooperatively by teams of classroom teachers and mathematicians. They were tried in classrooms and revised in the light of teachers' comments. The program is considered a complete one; all of the strands are incorporated. Emphasis on discovery of relationships and application of principles leads to a seemingly slower pace for mastery of some skills than is traditional. This procedure is followed in an attempt to avoid rote learning of mature "short cuts."

In the summer of 1962 the materials for Grades 4-8 will be revised and work on materials for primary grades will be started. In the fall of 1962 a five year evaluation program will be initiated.

To be placed on the mailing list for the Newsletter, send request to Dr. E. G. Begle, Director, School Mathematics Study Group, School of Education, Stanford University, Stanford, California.

Materials available for purchase:	Cost:
Mathematics for the Elementary School Textbooks for Grades 4, 5, and 6. (Preliminary edition) These are for examination purposes only. Three volumes per grade, one consisting of the textbook for pupils, the other two of the related commentary for teachers. 1962 revisions will be available for use with pupils. (See page 6.)	Per grade $ 2.00 pupils' book (1 vol.) 2.00 teachers' commentary (2 vol.)

Lists of units for each grade give an indication of the content. After study by a district, selected units could be very useful in supplementing in depth the content of a traditional program in both intermediate and upper grades. In-service help for teachers should be planned.

Grade 4. Concept of Sets* Number & operation
Numeration[2]
Properties and Techniques of Addition and Subtraction (I)[1]
Properties of Multiplication and Division[1]
Sets of Points
Properties and Techniques of Addition and Subtraction (II)[1]
Properties of Multiplication and Division[1]
Recognition of Common Geometric Figures
Linear Measurement
Concept of Fractional Numbers[1]

Grade 5. Extending Systems of Numeration[2]
Factors and Primes[1]
Extending Multiplication and Division[1]
Congruence of Common Geometric Figures
Addition and Subtraction of Fractional Numbers[1]
Measurement of Angles
Area
Ratio
Summary and Review

* Needed as background for most of the other units; is not recommended for use unless ideas will be utilized in work which is to follow.

1. Units numbered 1 are particularly useful in supplementing a traditional program by a new approach to developing understanding of operations and computational skills.

2. Units numbered 2 are useful in extending understanding of place value systems of numeration.

3. Units numbered 3 are useful in extending understanding of the number system to the integers and applications of these understandings in organizing data.

Grade 6. Exponents[2]
Multiplication of Fractional Numbers[1]
Side-Angle Relationships of Triangles
The Integers[3]
Coordinates[3]
Division of Fractional Numbers[1]
Volume
Organizing and Describing Data[3]
Summary and Review
Sets and Circles

1962 Edition

The 1962 revisions of the materials for Grades 4, 5, and 6 will become available for use in fall of 1962; minimum order to any purchaser will be a classroom set. These will be paper bound in 3 parts of pupils' material per grade level. The first part will be ready for delivery for use in September if ordered prior to June 1. Other parts will follow in time for use during the year. Estimated price per pupil set is $2.00. Commentaries will be provided for teachers. All SMSG materials are sold at cost.

Source:

A. C. Vroman, Inc.
367 South Pasadena Avenue
Pasadena, California
(Materials are distributed under a service contract.)

Mathematics for Junior High School (1962 revision)

This is considered a complete program for Grades 7 and 8. All strands are included. Chapter headings indicate the content of the pupils' texts. Selected sections could supplement a traditional program. Useful in in-service education. Note that supplements to this program are available also.

Volume I - 7th Grade

Chapter Headings:

1. What Is Mathematics?
2. Numeration
3. Whole Numbers
4. Non-Metric Geometry
5. Factoring and Primes
6. The Rational Number System
7. Measurement
8. Area, Volume, Weight, and Time
9. Ratios, Percents, and Decimals
10. Parallels, Parallelograms, Triangles, and Right Prisms
11. Circles
12. Mathematical Systems
13. Statistics and Graphs
14. Mathematics at Work in Science

Volume II - 8th Grade

Chapter Headings:

1. Rational Numbers and Coordinates
2. Equations
3. Scientific Notation, Decimals, and the Metric System
4. Drawings and Constructions
5. Symmetry, Congruence and the Pythagorean Property
6. Real Numbers
7. Permutations and Selections
8. Probability
9. Similar Triangles and Variation
10. Non-Metric Geometry
11. Volumes and Surface Areas
12. The Sphere
13. Relative Error

Publications available for purchase:	Cost:
Mathematics for Junior High School	
Vol. I, Student's Text, Parts I and II	$ 3.00 per set
Vol. I, Teacher's Commentary, Parts I and II	3.00 per set
Vol. II, Student's Text, Parts I and II	3.00 per set
Vol. II, Teacher's Commentary, Parts I and II	3.00 per set

(The two parts of any set cannot be ordered separately.)

Source:

Yale University Press
School Mathematics Study Group
92A Yale Station
New Haven, Connecticut

Junior High School Mathematics Units	Cost:
Vol. I, Number Systems, Student and Commentary	$ .70 ea.
Vol. II, Geometry, Student and Commentary	.60 ea.
Vol. III, Applications, Student and Commentary	.40 ea.
PKG (One each of the above 6 books)	2.95
Mathematics for Junior High School, Supplementary Units	
Student (JSV)	.80
Commentary (CJSV)	.65

Source:

A. C. Vroman, Inc.
367 South Pasadena Avenue
Pasadena, California

Supplementary Units (HS series), Studies in Mathematics (SM series)

The three supplementary units of the HS series are helpful to groups of teachers in study of number theory and the real number system.

The SM series for teachers includes eight publications dealing in some depth with topics from most of the strands. These series are mentioned here because of the wealth of background material they provide.

5. Stanford Projects

Two projects, often referred to as the Stanford Projects, were initiated by Professor Patrick Suppes and others connected with Stanford University. Both were initially financed by the National Science Foundation and the Carnegie Corporation.

a. Hawley, Newton and Suppes, Patrick. Geometry for Primary Grades.

First introduced in Grade 1, this experimental material now is intended for introduction to all pupils at the second or third grade level. The workbooks present geometric notions and constructions for the purpose of stimulating reasoning among pupils and developing the concept of precision. Considerable use of pencil, straight edge, and compass is required, and pupils are expected to read the instructions. It is not intended to replace arithmetic, and does not require knowledge of it. Books 3 and 4 are in preparation.

Materials available:	List Price*:
Geometry for Primary Grades, Book 1	$ 1.25
Geometry for Primary Grades, Book 2	1.25
Teacher's Manual for Book 1	.60
Teacher's Manual for Book 2	.60

20% discount on
orders of 5 or
more copies

Source:

Holden-Day, Inc.
728 Montgomery Street
San Francisco 11, California

b. Suppes, Patrick. Sets and Numbers.

The intent of this experimental material is to make arithmetic more meaningful and logical and to develop skills. The starting point is the idea of a set, and number is characterized as a property of a set. Considerable new symbolism is introduced in the early development. A complete program for Kindergarten through Grade 6 is being developed under the guidance of Robert E. K. Rourke, Consulting Editor. All of the strands are given some attention.

Materials available:	Suggested level of use:	List Price:
Book K1	Second semester of kindergarten	
Books 1A and 1B	Grade 1, standard course	
Book 1C	Grade 1, optional	
Book 2A and 2B	Grade 2, standard course	

Source:

Blaisdell Publishing Company
(A Division of Random House, Inc.)
22 East 51st Street
New York 22, New York

6. University of Illinois Arithmetic Project

These mimeographed materials were developed in connection with the University of Illinois Arithmetic Project which is financed primarily by the Carnegie Corporation. It is intended for teachers to use in learning to present basic mathematical ideas to pupils in an exciting and interesting fashion. The ideas are presented through many examples of classroom exercises and problems. No grade levels are given, but the author states that the materials have been used in first grade and up. The number line is emphasized throughout. Strands #1, Number Systems and Operations, and #3, Measurement, are developed extensively.

Materials available:	Cost:
Number Lines, Functions and Fundamental Topics	
Answers and Comments for the above	

Source:

David A. Page
University of Illinois Arithmetic Project
University of Illinois
1207 West Stoughton
Urbana, Illinois

7. University of Maryland Mathematics Project

This project for Grades 7 and 8 was begun in 1958 with the cooperation of the four major school systems in the area of Washington, D. C. It is financed by a grant from the Carnegie Corporation. One purpose is to determine maturity levels at which certain mathematical ideas can be taught appropriately and to prepare materials that are mathematically and psychologically sound. A complete sequence for Grades 7 and 8 has been prepared, tried in classrooms, and revised. The Advisory Board includes representatives of the U. S. Office of Education, the State Department of Education and various fields within the University; Dr. John R. Mayor is Project Director. Currently, programed materials are being used in learning studies. Problems of in-service education also are under study.

Direct inquiries to:

Mrs. Helen L. Garstens, Associate Director
University of Maryland Mathematics Project
University of Maryland
College Park, Maryland

Materials available for purchase:	Cost:
Mathematics for the Junior High School, Book 1	$ 3.00
Teacher's Guide, Book 1	2.00
Mathematics for Junior High School, Book 2	3.00
Teacher's Guide, Book 2	2.00

SUPPLEMENTARY MATERIALS FROM PUBLISHERS

No printed materials designed for supplementary use in K, 1, or 2 were submitted to the committee for study.

Grades 3 - 8

1. Exploring Mathematics on Your Own

Each booklet in this series is complete in itself. A booklet may be studied by a pupil individually or used by a group. The reading is relatively easy. Starred titles are suitable for supplementary use in Grades 7 and 8 in developing the strands which are indicated by the titles. The remaining booklets provide interesting enrichment material. Paper bound.

Materials reviewed:

Glenn, Johnson. Exploring Mathematics on Your Own.

		School Price
*1.	Sets, Sentences and Operations	$ .64
2.	The Pythagorean Theorem	.56
3.	Topology - the Rubber-Sheet Geometry	.52
*4.	Understanding Numeration Systems	.60
5.	Fun with Mathematics	.56
6.	Number Patterns	.56
7.	Invitation to Mathematics	.64
8.	The World of Statistics	.64
*9.	Short-Cuts in Computing	.56
10.	Computing Devices	.60
*11.	The World of Measurement	.64
12.	Adventures in Graphing	.64

Source:

Webster Publishing Company
367 South Pasadena Avenue
Pasadena, California

2. Ginn Arithmetic Enrichment Program

a. Three booklets of 96 pages each are provided for use in Grades 4-6. Each of the three includes ideas from all of the strands, with the exception that concepts of sets are not introduced until the third book. Much of the material is supplementary in that it introduces mathematical ideas, that most children have not previously encountered, in a form that makes easy the transition from the traditional to the new. Although in workbook form, the booklets are not necessarily consumable. They are usable with any series of textbooks. The booklet intended for Grade 4 might be used at a higher grade level with children who had not used it previously, and so on. Teacher's editions are available.

b. In Number Principles and Patterns, a nonconsumable booklet for upper grades, the author presents many one and two page units with appropriate questions. Mathematical ideas are introduced in a way which facilitates moving from a traditional to a contemporary program. The material is suitable for use in either Grade 7 or 8. While most of the units deal with ideas related to Number Systems and Operations, others of the eight strands appear. Paper bound.

Materials reviewed:	List Price*:
Marks, Smart, Brownell, Exploring Mathematical Ideas, 96 pp.	.64
Marks, Smart, Sauble, Enlarging Mathematical Ideas, 96 pp.	.64
Marks, Smart, Sauble, Extending Mathematical Ideas, 96 pp.	.64
Archer, Number Principles and Patterns, 68 pp.	.76

*Subject to discount

Source:

Ginn and Company
2550 Hanover Street
Palo Alto, California

3. Glenn, Mehl and Rasmussen. Using the Language of Algebra in Arithmetic, Exploratory Algebra

These two booklets are intended to supplement a traditional program. The first is very useful in Grades 6, 7, or 8. The Mathematical Sentence is emphasized to build facility in the "language of algebra" in very simple mathematical form. Considerable attention is given to Number Systems and Operations. The second booklet is designed to be an introduction to algebra, and the authors suggest that it be taught the semester preceding the regular course. Stress is on the strands mentioned above; Geometry, Measurement, and Graphs and Functions are given some attention. Teachers' editions are available. Paper bound.

Source:	Price:
Franklin Teaching Aids, Inc. 367 South Pasadena Avenue Pasadena, California	$ 1.25 each

4. Lay, L. Clark. Arithmetic - An Introduction to Mathematics.

While this book was originally planned for use in junior college or high school, the material can be used for supplementary purposes in Grades 7 and 8 for advanced groups. The Mathematical Sentence is emphasized throughout in problem solving and in stating identities; the content of Number Systems and Operations is fully developed. Material from other strands is limited. While ideas are stated concisely and clearly, pupils need assistance in reading material of this nature. There are many well planned sets of exercises. The book is useful in in-service education for transition programs. Hard bound.

Source:

The Macmillan Company
111 New Montgomery Street
San Francisco 5, California

List Price*:

$ 4.50

*Subject to discount

5. Larsen, Harold D. Enrichment Program for Arithmetic.

There is a set of eight different booklets for each grade, 3-8. Much of the material is recreational in nature; some is informative. The three booklets, Number Systems (7), Prime Numbers and Binary Systems (8) are useful for supplementary purposes in both intermediate and upper grades.

Source:

Row Peterson and Company
367 South Pasadena Avenue
Pasadena, California

List Price*:

$ 1.60 per set

*Subject to discount

6. Mathematics Enrichment, Programs A, B, C

These three programed books are suitable for use by individuals or by groups. Each contains three sections, Sets, Geometry and Numeration, that are sequentially developed from one book to the next. Though they are designated by grade, with Program A for Grade 4, they could be used at higher levels. They may be used to supplement any textbook. The books are scheduled to be ready for delivery in the summer of 1962. Response pads and "Skil-Slides" are to be used with them. Teacher's manuals are supplied without charge. The author is Dr. George Spooner of Central Connecticut State College. Hard bound.

Source:

Harcourt, Brace and World, Inc.
1885 Rollins Road
Burlingame, California

Price:

To be announced

(Estimated, approximately $3.00 per book, with accessory materials.)

7. Roche, Merrill. Number in the News Series.

Information from many newspaper and magazine articles was summarized to provide applications of mathematics in scientific settings in these three booklets of 20 pages each. Titles are Arithmetic in a Rocket Age, Arithmetic and Space Travel, and Arithmetic in a World of News. These are enrichment rather than supplementary material. Paper bound.

Source:

Franklin Teaching Aids, Inc.
367 South Pasadena Avenue
Pasadena, California

Price:

$1.35, packet of 3

8. Williams, Read, Williams. Modern Mathematics in the Elementary and Junior High Schools.

This work textbook of 158 pages includes many facets of Number Systems and Operations, Mathematical Sentences, Functions and Graphs, and Applications. The brief treatment of many of the topics leads to incomplete mathematical ideas and sophisticated presentation. It is useful in the classroom in the hands of a teacher who has considerable mathematical background and in in-service education. A teacher's edition is available. Paper bound.

Source:	List Price*:
L. W. Singer Company 249-259 West Erie Boulevard Syracuse 2, New York	$1.64 pupil's or teacher's *Subject to discount

9. Winston Extended Program Booklets

A major portion of each 64 page booklet is devoted to extension of the materials in the basic texts. Mathematical Sentences are used, and Number Systems and Operations are treated adequately. Many topics receive only cursory treatment. Teacher's guides are available, and workbooks are in preparation. Paper bound.

Materials reviewed:	List Price*:
Brueckner, Merton, Grossnickle. Extending the New Learning Numbers Extending the New Exploring Numbers Extending the New Understanding Numbers	$ .60 ea. *Subject to discount

Source:

Holt, Rinehart and Winston, Inc.
190 Waverly Drive
Pasadena, California

Report of the Subcommittee on Experimental Programs and Supplementary Materials of the California State Curriculum Commission's Advisory Committee on Mathematics
August, 1962

RECENTLY PUBLISHED MATERIALS AND PLANS BY PUBLISHERS FOR PUBLISHING MATERIALS PRIOR TO 1965

This list includes recently published materials as well as materials that will be available prior to 1965.

1. Addison-Wesley Company, 703 Welch Road, Palo Alto, California
   J. William Larkin, Jr., Director Elementary School Division,
   703 Welch Road, Palo Alto

   Completely modern K-8 mathematics series featuring Robert E. Eicholz as principal author; an extension of the Ball State--Greater Cleveland Mathematics Program. Texts for K, 1, and 2 to be available by January, 1963, with the remaining texts expected by the summer of 1964.

2. American Book Company, 55 Fifth Avenue, New York 3, New York
   Harry L. Kaiser, California Manager, 399 Adrian Road, Millbrae

   Completely new Grade 1-8 series in keeping with the essential integrating strands of mathematics authored by Deans, Kane, Osterle, McMeen. Texts for Grades 1, 2, 7, and 8 to be available by July, 1963. Remaining texts to be available by July, 1964.

3. Encyclopedia Britannica Films Inc., Wilmette, Illinois
   Phillip Gehman, California Manager, 1416 Wendell Way, Yuba City

   Wirtz and Botel, Math Workshop for Children, 1962.

   Individualized pupils' work texts each containing 128 or more looseleaf pages. Grade 1-4 texts are presently available with remaining Grade 5-6 texts anticipated shortly. Teachers' editions for Grade 1-2 texts are presently available with remaining editions anticipated shortly.

   The texts incorporate many of the essential strands of mathematics indicated by the Advisory Committee. Attractively illustrated with many new ideas for presenting the content. Contents of Grade 1 text: number and counting, equality and inequality, numeration, subtraction as the inverse operation of addition, division as inverse operation of multiplication, tables and number pairs, measurement, problem solving, and geometry. Publisher believes that the teachers' editions are self-instructive.

4. Harcourt, Brace, and World, 750 Third Avenue, New York 17, New York

   Clark, Junge, and Clark, One-By-One, Two-By-Two, 1962.

   Beautifully illustrated textbooks for Grades 1-2 with accompanying teacher's guides prepared in the pattern of most recent text-workbooks. (Texts for Grades 3-8 have also been recently revised according to many of the recent mathematical recommendations.)

5. Holt, Rinehart, and Winston, Inc., 190 Waverly Drive, Pasadena, California

   Brueckner, Merton, and Grossnickle, New Winston Arithmetics, Grades 1-8, revised 1959, 1962. Revision of the present state series with acceleration of content introduction. The teachers' guides contain additional teaching aids in keeping with some of the modern mathematical trends.

   New texts for Grades 7-8, under new authorship, are planned for 1963 and 1964.

6. Houghton Mifflin Company, 2 Park Street, Boston 7, Massachusetts

   Significantly revised Grade 1-2 texts anticipated by February, 1963. Grade 3-6 texts anticipated by 1964 or 1965.

7. Laidlaw Brothers, River Forest, Illinois
   Gordon Mackay, California Manager, 2121 Staunton Court, Palo Alto

   Thoburn, Nelson, and Gundlach, Arithmetic Primer; Arithmetic 1; Arithmetic 2; 1962.

   Well-illustrated K-2 text-workbooks and accompanying teachers' editions. This modern program fosters teacher confidence by a format similar to recent text-workbooks. The program develops increasing proficiency in computation and the ability to think in the language of arithmetic by using number words, numerals and operational symbols. The teachers' editions contain many helpful suggestions which help clarify and extend the teacher's understanding of modern mathematical concepts and explain how traditional activities fit into the framework of mathematics.

   Modern texts for Grades 3-8, written by Dr. Gundlach, Chief Consultant and Coordinator of the Greater Cleveland Program, are scheduled for production by February, 1963. Dr. Gundlach has also written The Laidlaw Glossary of Arithmetical-Mathematical Terms which was published by Laidlaw in 1961. This 120-page glossary, selling at 69 cents to schools, should be helpful to teachers in upgrading their arithmetical-mathematical vocabulary.

8. Prentice-Hall, Inc., Englewood Cliffs, New Jersey
   Earl Mott, California Manager, Monrovia

   Dawson, Numbers for You and Me, 1961.

   A supplementary text-workbook for use in the Kindergarten or Grade 1, designed with large pages and no written vocabulary. Suggested teacher's language includes the use of set, sequence, numeral, order, and the like. Zero is introduced early. (Modern texts for Grades 1-2 by Dawson are anticipated by March, 1963, Grades 3-6 by January, 1964. Modern Grade 7-8 texts by Dr. John R. Mayor, et al., are now available.)

9. Row, Peterson and Company, Evanston, Illinois
   Carl Dauler, California Manager, 117 Leona Court, Alamo

   Wheat, et al., Text-Workbook 1, Text-Workbook 2, 1962.

   Splendidly illustrated text-workbooks for Grades 1-2 prepared in the pattern of most recent text-workbooks. Teachers' editions contain many helpful materials and suggestions for teachers. (Revised texts for Grades 3-8 are anticipated by 1964.)

10. Scott, Foresman and Company, Chicago 11, Illinois
    J. R. Davenport, California Manager, 855 California Avenue, Palo Alto

    Revision of present Seeing Through Mathematics series anticipated by February, 1963. New Grade 7-8 texts, published in 1962, incorporate the recent experimental program in revised form.

11. Silver Burdett Company, Morristown, New Jersey

    No definite plans for revising, Making Sure of Arithmetic series, Grades 1-6.

    However, new Grade 7-8 texts, Modern Mathematics for Junior High School, Books 1 and 2, by Rosskopf, et al., were published in 1961.

12. Science Research Associates, Inc., 259 East Erie Street, Chicago 11
    Vernon Larson, California Manager, 3981 Fruitvale Avenue, Oakland

    SRA will publish shortly the revised Greater Cleveland Mathematics Program, Grades K-6. This series will be in keeping with most of the essential strands of mathematics recommendations. The K-3 materials are anticipated by August, 1962. The Grade 4-6 materials will probably be ready for use during the 1963-64 school year.

    SRA has recently published an interesting Math Discovery Board with accompanying teacher's handbook, designed for individual learning of elementary mathematical concepts. Strips of cardboard are used to represent numbers.

13. Van Nostrand Company, Inc., 120 Alexander Street, Princeton, New Jersey

    New Grade 7-8 texts are to be printed by February, 1963.

14. Webster Publishing Company, 1154 Reco Avenue, St. Louis 26, Missouri
    Hobard Jackson, Western Manager, 3155 Mulberry Avenue, South, Salem, Oregon

    Spitzer, et al., Exploring Arithmetic, revised series, 1961-1962.

    The significantly revised arithmetic series for Grades 1-8 makes extensive use of the number line. The teacher's editions contain many teaching suggestions.

ANNOTATED LIST OF MATHEMATICS MATERIALS

This list has been compiled to provide guidelines to assist teachers and administrators in modernizing course content and curricula in mathematics.

Many of the materials and much of the equipment listed below are described in greater detail in the following publications: Purchase Guide for Programs in Science, Mathematics, and Modern Foreign Languages, 1959, and the Supplement to Purchase Guide for Programs in Science, Mathematics, and Modern Foreign Languages, 1961. These guides have been prepared by the Council of Chief State School Officers with the assistance of Educational Facilities Laboratories, Inc. We would strongly recommend these sources for descriptions and specifications of mathematics materials and equipment. Copies are available in both the Professional Library of the City Schools' Library, San Diego Unified School District; and the Curriculum Library, Department of Education, San Diego County.

ABACUS, COMPUTING

A manipulative device to aid elementary school pupils in developing an understanding of number concepts, verifying arithmetic processes, etc.

ABACUS, STANDARD BEAD

To aid in teaching young children to count, group and discover arithmetic facts. Especially useful with slow learners.

ARITHMETIC, MANIPULATIVE TEACHING DEVICES

Manipulative teaching devices are used for developing number concepts, for illustrating the operations of arithmetic, the representation of numbers to various bases, for demonstrating various concepts of measurement (English and metric systems).

BLOCK, CUBIC FOOT, DISSECTIBLE

This block is used to teach concepts of linear, area, and volume relationships in the English system of measurement. The block is dissectible into component parts of different volume. Children can thus visually develop concepts of length, area, and volume and derive the basic formulas for computing perimeters, areas, and volumes. The various component parts can be immersed in water so that displacement tests of volume can be made.

BLOCKS, STERN

Multicolored manipulative blocks of varying lengths widely used to reinforce mathematics content in the primary grades.

BOARD, PLACE VALUE

A manipulative device resembling a modern abacus for use in the intermediate and upper elementary grades to permit students to picture and read numbers to billions, and decimals to ten-thousandths.

CHART, PLACE VALUE

For teaching elementary school pupils the position of ones, tens, and hundreds in our decimal system. In using the device, students place cards on hooks in such a way as to show structure of the number system, and to visualize the complete operation and regrouping required in the addition, subtraction, multiplication, and division processes.

CLOCK DIAL

To aid in teaching primary pupils how to tell time.

COMPASS, CHALK BOARD

For compass work with chalk on a blackboard.

COUNTING FRAME, DEMONSTRATION SET

An aid in teaching the meaning of the number system, and to visualize number combinations, including those of higher decades.

COUNTING FRAME, PUPIL USE

A visual aid to help pupils develop meaning and understanding of the structure of the number system. A convenient device for teaching pupils to visualize number combinations, including those of higher decades.

CUBIC FOOT

To provide a visual illustration of the size of a cubic foot.

FIGURES, PLANE GEOMETRY, DEMONSTRATION SET

Series of manipulative sensory aids for demonstrating the basic principles of plane geometry and to guide the students' learning by induction. An aid in discovering relationships intuitively.

FLASH CARDS

Used to familiarize elementary school pupils with basic number combinations in addition, subtraction, multiplication, and division. Especially useful in drill work.

FRACTION DISCS

To aid in teaching the meaning of fractions, fractional relationships, etc.

GEOMETRIC SOLIDS

To show many of the principal geometrical solids, surfaces, and angles discussed in elementary and secondary school mathematics.

GEOMETRY, RULER SET, BLACKBOARD STENCILS

Permit easy drawing of erasable plane geometric figures on chalkboard for classroom work. The stencils can be held against the blackboard, details traced with chalk, and measurements made with the ruler and figure numbers. Instructions for use shall be provided. Also supplied, when specified, shall be suction cups to prevent slipping when it is desired to maintain the stencil stationary for a period of time.

GEOMETRY RULER SET, STUDENT

A ruler-diagram device for speedy drawing of common geometric figures, and for teaching students the proportions, shapes, and mathematics of these geometric figures.

GRAPH CHART, RECTANGULAR COORDINATE

An erasable rectangular coordinate chart for individual or group plotting of data.

GRAPH PAPER, RECTANGULAR COORDINATE .

To plot mathematical and statistical data.

KITS

Kits are available in arithmetic, for example, to develop number concepts. These kits are usually accompanied by an instruction manual. The chief advantage of this type of kit is that it includes in one package the equipment needed to help students develop a number of basic concepts in a science or mathematics area.

The following kits are included in the Master List:

Computing Machine Construction Kit
Drawing Kit

MEASURES, LIQUID, ENGLISH

Used to show the relationship between the various English (liquid) units of measure.

MEASURES, LIQUID, METRIC

Used to show the relationships between the various metric (liquid) units of measure.

METER STICKS

Reversible scales on either side permit measurements to be made in yards or meters, as well as to demonstrate the relationships between the English and metric units of length.

MULTIPLICATION AND DIVISION KIT

To aid in teaching groupings in multiplication and pupil discovery of multiplication and division facts.

NUMBER BUILDER

Used to familiarize the pupil with combinations in addition, subtraction, multiplication, division, and simple fractions. Also useful for review or drill purposes. The student uses the set in this manner: He selects at random a problem card on which a number combination appears and then places it under the correct answer card lying before him on the table. This process is repeated with other problem cards.

PATTERN DIAL

The pattern dial is used to draw regular polygons accurately and also to serve as a precision protractor and compass.

PEGBOARD AND PEGS, INDIVIDUAL

A visual aid in teaching elementary pupils to count, add, and multiply.

PROTRACTOR, CHALKBOARD, REGULAR
To facilitate chalkboard work involving the measurement of angles.

RODS, CUISENAIRE
Groups of colored rods ranging from one to ten centimeters. Devised to give concrete form to abstract arithmetic.

RULER, ENGLISH AND METRIC SCALES
For use in drawings and measurements in the usual mathematics classwork.

TAPE, MEASURING, METRIC AND ENGLISH
To measure large objects, distances, heights, etc. Linen tapes are often preferred to steel tapes--in the elementary school--because they are less fragile.

TAPE, MEASURING, STEEL, DECIMALIZED
To make various direct measurements in simple surveying and other projects where accuracy is limited to the nearest 0.01 feet.

THERMOMETER, LARGE, WALL
This is a large wall thermometer for use in the primary grades to help young children learn how to read a thermometer and to be introduced to the concept of degrees and to centigrade and Fahrenheit scales.

TIMER, CLOCK, ELECTRIC, STOP
Laboratory clock for timing continuous or interrupted intervals for individual or small group work.

TRIANGLE, CHALKBOARD
To facilitate the drawing of lines and angles on the chalkboard.

T-SQUARE, CHALKBOARD
For chalkboard use in drawing straight lines and right angles.

SELECTED LIST OF FILMS AND FILMSTRIPS IN MATHEMATICS

Many of the materials listed on the following pages closely parallel the content of the Television Mathematics Series. Other films and filmstrips are included to assist you in planning a better mathematics instructional program.

I. Films, Filmstrips, and Tapes Available from the Audio-Visual Section, San Diego County Department of Education

Primary Grades

Title	Catalog No.	Color	Minutes
FRACTIONS, COMMON			
Parts of Things	MP 1241	b/w	10
GROUPING PROCESSES			
Adding and Subtracting	MP 1953	color	10
Adding and Subtracting Two Place Numbers	FS 2927	color	30 fr
Addition Is Easy	MP 1799	b/w	10
Arithmetic for Beginners, Part 1	MP 1787	b/w	5
Arithmetic for Beginners, Part 2	MP 1788	b/w	8
Arithmetic for Beginners, Part 3	MP 1789	b/w	7
Column Addition	FS 2924	color	30 fr
Compound Subtraction	FS 1286	color	
Duck Eggs	FS 607	color	33 fr
Number Family in Addition	FS 2063	color	
Number Stories of 11 and 12	FS 2928	color	30 fr
Number Stories of 9 and 10	FS 2923	color	30 fr
Number Stories of 7 and 8	FS 2922	color	30 fr
Number Stories of 13 to 18	FS 2929	color	30 fr
Parts of Nine	MP 1240	b/w	11
Roy's Number Friends	FS 609	color	32 fr
Sign Language	FS 2920	color	30 fr
Storekeepers	FS 610	color	34 fr
Subtract To Find Out	FS 2925	color	30 fr
Tea Party	FS 606	color	30 fr
The Threes	FS 2518	color	44 fr
Twos in Division	FS 2555	color	49 fr
What Is Four?	MP 5161	b/w	15
LINEAR MEASUREMENT			
Let's Measure: Inches, Feet and Yards	MP 1623	color	10
NUMBER CONCEPTS			
Circus	FS 608	color	28 fr
Counting to 20 by 1's, 2's, and 5's	FS 2921	color	30 fr
Getting Acquainted with Numbers	FS 601	color	29 fr
How Many?	FS 602	color	26 fr
Let's Count	MP 1306	b/w	10
Let's Play Numbers	FS 603	color	29 fr

Title	Catalog No.	Color	Minutes
Number Concepts cont'd.			
Pedro and His Sheep	FS 600	color	29 fr
Recognizing Numbers	FS 604	color	28 fr
Taller? Shorter? Larger? Smaller?	FS 605	color	28 fr
Teen Numbers	MP 1243	b/w	10
Understanding Hundreds, Tens, and Ones	FS 2926	color	30 fr
What Is Four?	MP 5161	b/w	15
What Numbers Mean	FS 2656	color	
Zero, A Place Holder	FS 2724	color	
TIME			
Calendar: Days, Weeks, Months	MP 1668	b/w	10
Calendar Tells When	FS 1190	color	26 fr
How To Tell Time	FS 1727	b/w	
How We Tell Time	FS 1730	color	
Telling Time by the Clock	MP 1797	b/w	10
Time for Everything	FS 2520	color	
LATEST CATALOG ADDITIONS			
Abacus	MP 2090	color	11
Addition by Regrouping and Renaming	FS 665		
Addition Through Nine	FS 641		
The Basic Addition Table	FS 652		
Computers, Numbers, and People	FSR 40		
Development of the Subtraction Algorism	FS 675		
Division Algorism	FS 679		
Expanded Notation	FS 672		
Fractions	FS 650		
From Addition to Subtraction	FS 673		
Greater Numbers and Magic Squares	FS 676		
Introducing Division	FS 678		
Introduction to Multiplication	FS 668		
Key Number Ten	FS 664		
The Language of Sets	FS 647		
Mathematics at Your Fingertips (Teacher Education)	MP 6237		
Multiplication	FS 677		
Number Games--Fractions and Measure	FS 680		
The Number Line	FS 651		
Playing Number Games	FS 669		
Playing with Numbers	FS 648		
Review and Extension of Fundamentals	FS 663		
Review of Sets	FS 671		
Subtraction	FS 674		
Subtraction Algorism	FS 667		
Subtraction and the Key Number Ten	FS 666		
Subtraction Through Nine	FS 653		
The Understanding of Ten	FS 646		
Working with Fractions: Measuring Time and Space	FS 670		

Intermediate Grades

Title	Catalog No.	Color	Minutes
ADDITION AND SUBTRACTION			
History of Our Number System	FS 1647	b/w	32 fr
AREA			
History of Area Measures	FS 1083	b/w	29 fr
CHARTS, GRAPHS, AND TABLES			
Language of Graphs	MP 5162	b/w	13
FRACTIONS, COMMON			
How To Add Fractions	MP 1178	color	11
How To Change Fractions	MP 1231	color	11
How To Divide Fractions	MP 1438	b/w	11
How To Multiply Fractions	MP 1232	color	11
How To Subtract Fractions	MP 1225	color	11
Introduction to Fractions	MP 1174	color	11
What Are Fractions?	MP 1348	b/w	10
FRACTIONS, DECIMAL			
Meaning of Percentage	MP 1239	b/w	10
Percent in Everyday Life	MP 1424	color	10
Percentage	MP 1440	b/w	11
What Are Decimals?	MP 1511	b/w	10
LINEAR MEASURE			
Assignment: Weights and Measures	MP 5769	color	20
History of Area Measures	FS 1083	b/w	29 fr
History of Linear Measures	FS 1644	b/w	35 fr
MONEY			
Fred Meets a Bank	MP 1437	b/w	10
Science and Money	T 1329		15
Story of a Dollar	T 1414		15
What Is Money?	MP 1442	b/w	10
TIME			
Adventure with Time	T 1395		15
Clock of the World	T 219		30
Day and Night	T 174		30
Father Time, Himself	T 1483		15
History of Our Calendar	FS 1645	b/w	34 fr
History of Telling Time	FS 1648	b/w	37 fr
International Date Line	FS 1765	b/w	39 fr
Keeping Correct Time	FS 3103	color	29 fr
Science of Telling Time	T 1330		15
Story of Time	MP 1471	color	10
VOLUME			
History of Weight and Volume Measures	FS 1650	b/w	35 fr

Title	Catalog No.	Color	Minutes
WEIGHT			
Assignment: Weights and Measures	MP 5769	color	20
History of Weight and Volume Measures	FS 1650	b/w	35 fr
LATEST CATALOG ADDITIONS			
Abacus	MP 2090	color	11
Arithmetic: Understanding the Problem	MP 2110	b/w	11
Computers, Numbers, and People	FSR 40		
Mathematics at Your Fingertips (Teacher Education)	MP 6237		
Measuring Areas, Squares, Rectangles	MP 2111	b/w	11

II. Films and Filmstrips Available from the Audio-Visual Department, San Diego Unified School District

Films

Title	Min.	Color	Level
How Far?	10	color	Kdgn. - Pri.
Which Way?	10	color	Kdgn. - Pri.
The Calendar: Days, Weeks, Months	10	b/w	Pri.
What Time Is It?	10	b/w	Pri.
Numbers for Beginners	11	color	Pri.
Telling Time by the Clock	11	b/w	Pri.
What Is Four?	15	b/w	Pri.
Adding Is Easy	10	b/w	Pri. - Int.
Adding and Subtracting	10	color	Pri. - Int.
Parts of Things	10	b/w	Pri. - Int.
Subtracting Is Easy	10	b/w	Pri. - Int.
Story of Weights and Measures	10	b/w	Int.
Dividing Is Easy	10	b/w	Int.
Parts of Nine	10	b/w	Int.
Decimals Are Easy	10	b/w	Int.
Abacus	11	color	Int.
Decimal Fractions	11	color	Int.
How To Add Fractions	11	b/w	Int.
How To Change Fractions	11	b/w	Int.
How To Divide Fractions	11	color	Int.
How To Multiply Fractions	11	b/w	Int.
How To Subtract Fractions	11	b/w	Int.
Introducing Fractions	11	b/w	Int.
We Discover Fractions	11	b/w	Int.
Measurement	10	b/w	Int. - Jr.
Meaning of Percentage	10	b/w	Int. or Jr.

Filmstrips

Title	Order No.	Color	Level
Advancing in Quantity Measurements	Bl 511	color	Int.
Advancing in Simple Addition	Bl 511	color	Pri.
Advancing in Simple Subtraction	Bl 511	color	Pri.
Arithmetic for Beginners, Series I Number Readiness	Bl 511	color	Pri.
Arithmetic for Beginners, Series II Number Concepts	Bl 511	color	Pri.
Count 1, 2, 3, 4, 5	Bl 511	color	Pri.
Experiences with Comparisons	Bl 511	color	Pri.
History of Area Measures	Bl 511	b/w	Int. - Jr.
History of Linear Measures	Bl 511	b/w	Int. - Jr.
History of Our Number System	Bl 511	b/w	Int.
History of Weight and Volume Measures	Bl 511	b/w	Int. - Jr.
Making Change	Bl 511	color	Int.

Filmstrips cont'd.

Title	Order No.	Color	Level
Threes	Bl 511	color	Pri.
Time and Money	Bl 511	color	Pri.
Using and Understanding Division, Objects and Symbols	Bl 511	color	Pri.
Using and Understanding Multiplication, Objects and Symbols	Bl 511	color	Pri.
Using and Understanding Numbers by 2s, 3s, 4s, and 5s	Bl 511	color	Pri.
Using and Understanding Numbers Series Grade One	Bl 511	color	Pri.
Using and Understanding Numbers Series Kindergarten and Grade One	Bl 511	color	Kdgn. - Pr
Addition and Subtraction, Part I	Bl 511.2	color	Pri.
Addition and Subtraction, Part II	Bl 511.2	color	Int.
Multiplication and Division	Bl 511.2	color	Int.
Reading and Writing Whole Numbers	Bl 511.2	color	Pri.
Solving Problems	Bl 511.2	color	Int.
What Are Numbers?	Bl 511.2	color	Pri.
Addition of Fractions	Bl 511.4	color	Int.
Adding Decimal Numbers	Bl 511.4	color	Int.
Adding with Fractions	Bl 511.4	color	Int.
Beginning to Multiply and Divide with Fractions	Bl 511.4	color	Int.
Decimal Fractions: Division	Bl 511.4	color	Int.
Decimal Fractions: Hundredths	Bl 511.4	color	Int.
Decimal Fractions: Multiplication	Bl 511.4	color	Int.
Decimal Fractions: Subtraction	Bl 511.4	color	Int.
Decimal Fractions: Tenths	Bl 511.4	color	Int.
Decimal Whole Numbers	Bl 511.4	color	Int.
Different Names for Same Fractional Numbers	Bl 511.4	color	Int.
Dividing with Fractions	Bl 511.4	color	Int.
Division of Fractions	Bl 511.4	color	Int.
Experiences with Fractions	Bl 511.4	color	Pri.
Extending Division and Multiplication of Decimal Fractions	Bl 511.4	color	Int.
Language of Fractions	Bl 511.4	color	Int.
Multiplication of Fractions	Bl 511.4	color	Int.
Multiplying with Fractions	Bl 511.4	color	Int.
Origin and Meaning of Fractions	Bl 511.4	color	Int.
Subtraction of Fractions	Bl 511.4	color	Int.
Subtracting with Fractions	Bl 511.4	color	Int.
Uses of Fractions	Bl 511.4	color	Int.
What Is a Fraction?	Bl 511.4	color	Int.
Working with Equal Fractions and Reducing Fractions	Bl 511.4	color	Int.
Working with Like Fractions and Improper Fractions	Bl 511.4	color	Int.
Introduction to Percentage	Bl 511.8	color	Int.

A complete listing of all films and filmstrips can be found in the catalogs and in the master file in the Audio-Visual Department, San Diego Unified School District.

Filmstrips directly related to the new mathematics program are being cataloged and will be available shortly:

The Language of Sets
The Basic Addition Table
The Understanding of 10
The Number Line
Addition Through 9
Subtraction Through 9
Playing with Numbers
Fractions

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B. Curriculum Library
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Note: In addition to the references cited here, the materials listed in *Experimental Programs and Supplementary Materials, An Annotated Bibliography* (Item 3 in this packet) are also available.

II. References Available from the San Diego Unified School District

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__________. Studies in Mathematics, Volume 6: Number Systems. Stanford University, 1961.

Swain, Robert L. Understanding Arithmetic. Holt, Rinehart, and Winston, 1957.

Taylor, E. H. and Mills, C. N. Arithmetic for Teacher-Training Classes. 4th ed. Holt, Rinehart, and Winston, 1955.

Recent issues of the following magazines contain articles on contemporary mathematics:

American Mathematical Monthly

Arithmetic Teacher

Mathematics Magazine

Mathematics Teacher

Recreational Mathematics Magazine

School Science and Mathematics

Updating Mathematics

Note: In addition to the references cited here, the materials listed in Experimental Programs and Supplementary Materials, An Annotated Bibliography (Item 3 in this packet) are also available.

CRITERIA FOR EVALUATING BASIC TEXTBOOKS IN MATHEMATICS FOR GRADES ONE TO EIGHT

(Adopted by the State Curriculum Commission, May 15, 1962)

I. SCOPE OF CONTENT

A. Problem Solving and Application

The series of texts shall provide for:

1. Problems which are presented in relationship to processes
2. Problems which are distributed so as to develop and maintain skills in different types of problem solving
3. Problems which are accurate in content and realistic for the age group for which the book is intended
4. Problems which are arranged in gradually increasing order of difficulty
5. Problems which provide adequate content dealing with several types of arithmetic concepts which develop and maintain desired understandings and skills

B. Grade Placement and Sequence

The series of texts shall provide for:

1. Content for each grade which is adapted to the needs, interests, and abilities of pupils
2. The introduction of new processes and skills at equitably distributed intervals throughout all grades
3. Gradation of materials which reflect the current realization that elementary pupils are capable of learning more mathematical content than has been previously introduced
4. A balance among the meaning of the number system, computation, and application and a graduated approach to the structural properties of arithmetic

C. Nature of Number Systems

The series of texts shall provide for:

1. Matching or one-to-one correspondence as the basis for the pupil's instruction in learning to count
2. The definition of operations and their relationships
   a. Subtraction as the inverse of addition
   b. Division as the inverse of multiplication
3. The properties of the operations such as: Commutativity, associativity, distributivity, closure, and identity presented in terminology appropriate for elementary pupils

4. The correct use of the terms "number" and "numeral" and symbols appropriate to the ability level of elementary pupils

5. The number line as a visualization for mathematical abstractions

6. Sufficient development of the multiplicative structure of numbers, including the study of factoring and prime numbers

7. Rate, ratio, and per cent presented as special interpretations of rational and real numbers

D. Systems of Numeration and Notation

The series of texts shall provide for:

1. A thorough development of place-value and the decimal system, including the meaning of exponents

2. A consideration of systems of numeration with bases other than ten

3. A study of some historical bases, including Roman numerals, which led to the discovery and utilization of the decimal system

E. Geometry

The series of texts shall provide for:

1. An introduction to the simpler plane configurations and their use in life situations

2. An understanding of the simpler relationships among geometric configurations

3. The construction of simple geometric drawings and models

4. Opportunities to recognize common plane and solid figures and their distinguishing characteristics

F. Measurement

The series of texts shall provide for:

1. Development of the concept of measurement as an association of a number with an object through comparison with an arbitrarily chosen unit

2. A study of common standard units, including those of the metric system

3. Opportunities to convert measures to different units

4. Instruction in estimating in measurement

5. Computation with approximate data including the ideas of precision and accuracy

6. Drawing to scale

G. Mathematical Sentences
The series of texts shall provide for:

1. The use of mathematical sentences in the presentation of number facts

2. The use of mathematical sentences in problem solving

3. The use of mathematical sentences as an aid in stating mathematical principles precisely

H. Graphs and Scale Drawings
The series of texts shall provide for:

1. Graphs and scale drawings presented in such a manner that the pupils understand the mathematical significance of them

2. The use of graphs and scale drawings in practical, life-like situations

3. Graphs and scale drawings which are within the range of interest and understanding of elementary pupils

I. Logic

The series of texts shall provide for an informal treatment, at a level that has meaning for pupils, of such concepts of logical thinking as definition, implication, and quantification

J. Sets
The series of texts shall provide for:

1. An informal introduction of the set concept as a means of securing more precise definitions in mathematics

2. An introduction to the terminology and notation of sets appropriate to the grade level

II. MANNER OF PRESENTATION
The series of texts shall provide for:

A. Pupil involvement and pupil exploration leading to discovery of mathemetical principles and procedures

B. The introduction of processes in meaningful situations that will be clear and obvious to elementary pupils:

1. The pupils should be led through a sequence of steps or questions which result in discovery rather than to follow a prescribed series of directions.

2. Each step involved in a process should be well explained, providing at all times for the pupil to discover reasons for the step.

3. Manipulative and illustrative material should be suggested and/or included to help pupils discover and understand each new process.

4. The algorisms of arithmetic should be presented in a mathematically meaningful manner.

C. Opportunities for pupils to discover different ways of performing an operation

D. The introduction of arithmetical terms with adequate explanation. The terms should be used consistently throughout the text.

E. Opportunities for pupils to experiment and discover many ways to make reasonable estimates of correct answers

F. Opportunities for pupils to develop effective approaches to problem solving

G. Opportunities for pupils to check their own work for accuracy

H. Experiences for mental computation

III. TEACHER'S EDITION
A teacher's edition is essential. It shall include:

A. The pupils' text material and teacher aids

B. Statement of purpose and objectives for each lesson

C. Suggestions for concrete representation when appropriate

D. Adequate information concerning the mathematical background underlying any given lesson

E. Suggestions for presenting each lesson

F. Suggestions for providing for individual differences

G. Suggestions for evaluating pupil progress

H. An index

IV. READABILITY
Each book of the series of texts shall provide for:

A. A vocabulary which is suited to the grade level for which it is intended

B. A sentence structure that is well adapted for understanding by pupils of the grade level for which it is intended

C. A clear, concise explanation of mathematical terminology

V. PROVISION FOR INDIVIDUAL DIFFERENCES
The series of texts shall provide

A. Activities designed to meet the normal range of ability within a class

B. Exercises and problems within a lesson which are graduated toward increasing difficulty

C. Activities which reinforce concepts developed for the slower learner

D. Activities designed to challenge the more capable learner

VI. PRACTICE MATERIALS AND MAINTENANCE
The series of texts shall provide for:

A. Practice material distributed according to known learning difficulties

B. Sufficient practice for initial mastery

C. Practice materials properly distributed for maintenance and reteaching

D. Sufficient variety of practice materials to stimulate interest

VII. EVALUATION AND RETEACHING
The series of texts shall provide for:

A. Ample evaluating materials for each aspect of instruction

B. Periodic testing of large units

C. Continued reinforcement of learning and thorough review

D. Diagnostic tests and exercises which point to specific instructional needs and which may refer to content of previous grades

E. Practical self-checking devices for pupils

F. Tests and remedial materials which are designed for practical and economical use by teachers and pupils

VIII. PHYSICAL FEATURES
The series of texts shall have:

A. Format

1. Appropriate spacing and arrangement of pages

2. Illustrations which enhance the learning process and emphasize mathematical concepts

3. Captions to aid learning and to contribute to the teachability of the text

4. Type size and style suitable to grade level

5. Color appropriately used to aid in teaching and learning

B. Index, Table of Contents, Glossary, References